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Optimal planning of fixed route bus transit systems : a systems approach

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OPTIMAL PLANNING OF FIXED ROUTE BUS TRANSIT SYSTEMS : A SYSTEMS APPROACH

BY

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ABSTRACT

A new bus transit planning tool is developed for application in determining operating policies of a fixed-route bus transit system. The objective of the study is to model a bus transit system which functions under time-varying passenger demands and service characteristics.

The two phase transit model developed in this research is intended for use as a mass transit planning tool, to solve transit problems confronting the mass transit planner. The model is used to compute cost differentials in transit system options. These alternatives of expanding, abandoning or modifying service depend upon the service frequency, fleet size and other system attributes such as operating speed, delay, passenger demand and relevant cost factors.

The model is formulated in two phases, jointly utilizing linear and dynamic programming techniques. It is directed toward optimizing transit operation during one period and then aggregating each operation over the range of transit service periods. The basic components of system function to be optimized (minimal total cost) include such variables as bus operating and ownership costs, passenger costs in terms of walking, riding, and transfer times as well as bus fares.

The transit model has been programmed for a digital computer. This model requires inputs of existing street configuration and bus

routes, bus schedules, speed and delay data for street networks, fare structure, load factor and passenger Origin-Destination information for different periods.

A practical application of the transit model is presented in the format of a case study. This application illustrates the utilization of the methodology for deriving bus transit operating policies and the consideration of planning alternatives. The result of a comparison of these policies and alternatives is a significant reduction in the total system cost.

Special emphasis has been given to the analysis of the structural elements involved in a transit system as well as new transit planning techniques. There follows a summation of the findings and the implications of the results. This summary includes an appraisal of the model as to its limitations as well as recommendations for future research. The appendix, finally, lists a summary of notations, review of previous research, flow charts and listings of computer programs, supplemental data, computer input and output files, and an annotated bibliography containing current literature concerning the operation and planning of public transportation.

APPROVAL OF DISSERTATION

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TRANSIT SYSTEMS : A SYSTEMS APPROACH

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YOUNG LEE

FOR

DEPARTMENT OF CIVIL AND ENVIRONMENTAL ENGINEERING

BY

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CHAPTER I

INTRODUCTION

This thesis is a systems analysis of bus transit service in urban areas. The study involves the structuring and modeling of a bus transit system to develop an analytical planning tool which transit planners can utilize to determine where, when and how to improve fixed route bus transit service in congested urban centers. For this purpose, an analytical transit model is developed and tested to measure the performance of a typical fixed route bus transit system.

A fixed route service refers to a bus route which is established on well defined street links and does not change between schedule periods. Buses on a fixed route run according to a printed transit time table as opposed to taxi or dial-a-bus systems which have greater flexibility in the selection of a route and operating time.

The study is conducted from the systems viewpoint to reflect the effects of various bus system components upon both transit users and operators. The approach of the study is first to derive an optimum bus transit operation during one schedule period and then to aggregrate those transit operations throughout various schedule periods for an overall optimum system configuration.

This study is oriented toward the use and need of the public transportation planning agency and the local municipal government. The public agencies charged with the responsibility of planning and operating transit systems need to view bus transit service in the broad perspective of its benefits and costs to the community.

Bus Transit as a Public Service

In recent years there has been a growing desire for improved public transportation services throughout the country. This is especially true in urbanized areas where higher population densities provide sufficient public transportation users to support a transit system. The regional services such as industry, retail business, education, and health care provided in these urban centers are mainly supported by available public transit services for their functioning.

In urban areas, people depend on public transportation for work, recreation, and other social activities because of congestion, parking problems and various other constraints to private automobiles.

The present auto-based transportation system does not meet the needs of people who are left to use the transit system. These "captive" riders, the elderly, the poor, the handicapped and the young, suffer serious disadvantages from being served improperly. The proportion of captive riders is growing higher in urbanized areas and there is a definite need for improving mass transit systems to increase the mobility of such people.

For example, in the study area for this thesis, Newark, New Jersey, 52 percent[1] of the trips to and from the Central Business District are by mass transit.[2] For local trips within the City, the percentage is even higher, with 57 percent of the trips by transit. A recently completed bus survey found that of the total bus ridership on selected bus lines, two-thirds are captive riders having no other means of transportation.[3]

The Tri-State Transportation Commission's Home Interview Survey[4] in 1964 found that an estimated 72,000 passengers use public transportation for a one way trip daily in Newark. The 1969 Newark bus transit study[5] reveals that 34 bus companies are operating an estimated 2,945 buses in the Newark area. The highest daily volume of 1,979 buses in one direction occurs northbound on Broad Street between Clinton and Commerce Street, which indicates the magnitude of bus usage. The trips by transit are predominantly work oriented, with concentrations in two peak periods (6 A.M. - 9 A.M. and 3 P.M. - 6 P.M.).

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[1]See (68) P. 2.

[2]Mass transit means "Transportation serving the general public and moving over prescribed routes" U.S. Public Law 88-365. Mass transit generally refers to urban bus and rail service.

[3]For more information on captive riders, see Deutschman (78).

[4]For summarized daily transit trip from Central Business Districts in Newark metropolitan area, see (68) Table 4.

[5]For more information, see (105).

A large segment of the population is dependent upon mass transit as evidenced by the magnitude of transit service provided in the study area. Consequently, the access to urban opportunity and the economic vitality of urban centers such as Newark are almost entirely dependent on the availability of public transportation. The major portion of the public transportation in states such as New Jersey is provided by bus systems, which carry more than nine times as many people as are carried on the rail system.[6]

Buses, as a mode of mass transit, have advantages over rail transit. One advantage is the flexibility of bus transit system in coping with problems which are presently affecting many core cities in urban areas - such as the shift of population and industry which generate shifting patterns of travel demands. Another advantage of the bus system is its effectiveness in serving a lower level of demand with less capital investment than rail transit. Rail transit is feasible only in relatively few areas of extremely high population density.

Consequently, mass transportation solutions in most urban areas look to bus transit systems. However, present bus service is characterized by the long walk to the bus stop, frequent delays

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[6]See (68) P. 1.

to load and discharge, low operating speed, inflexible routes, infrequent service, multiple transfers, no shelter for inclement weather, lack of service information and high fares. Total on-bus time of 35 minutes to travel less than three miles of urban arterial in Newark highlights the inefficiency of the system.[7]

Need for Improved Planning Technology

Bus transit may be successful when it uses its inherent flexibilities to best serve movements in congested urban areas. However, transit operators are reluctant to provide new service or to change system components largely because of the lack of planning tool which can efficiently test alternative bus transit service configurations before they are actually implemented on the street network.

The complexity of bus networks, systems parameters and the multiple demand patterns with high peaking characteristics within an urban area make it difficult to assess the measure of major transit system outputs such as revenue, passenger benefits, transit operating costs and the return of system improvements.

The development of a transit model for determining cost-utility of transit operations and the optimal planning of bus service is

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[7]For more information on travel speed, delay time and service time, see (104).

highly desirable due to the magnitude of analysis involved and the far reaching effects of the system modifications. A planning model can help to redesign existing routes and to provide more direct and convenient trips.

A need for improvement of route systems is generally recognized by transit management. Nonetheless, the steadily decreasing patronage of bus transit and increasing labor and equipment costs make it difficult to justify expenditures for analyzing route system and scheduling practice on a continuous basis. The existing manual process of constructing new routes and a schedule policy based on a schedule maker's subjective judgement is very time consuming and expensive but still does not provide information on the optimal solution.

Therefore, to overcome the limitation of the manual method and to take account of the effect of the relocation of the transit user market and the shift of travel patterns, the necessity of developing a planning model for bus system analysis is realized.

Furthermore, some public aid will be necessary to augment the transit revenue obtained from passenger fares. For this purpose, the Federal Mass Transit Act was enacted to finance the capital improvements of mass transit systems.[8]

In this regard, the questions to be considered are to determine what form and what amount of public assistance is needed to satisfy the transit requirements for the optimal operation or, in the worst situation, just for the survival of the existing bus transit system. To answer these questions, a validated transit model as posed in this research is necessary.

The proposed functions of the transit model are not only to evaluate the need in the order of improvement priority, but also to determine the necessary amount of service to be retained. Another function of the model is to take proper accounts of all costs that incurred to both transit users and operators.

Improvements of transit service for each service period and route should be ordered based on the urgency of need. For example, one bus route in the system may have a higher priorty than another route because of a greater concentration of passengers. Likewise, one period, the weekday morning rush hour, may need more bus vehicles than the Sunday period.

The determination of amount of service requires special consideration

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[8]In fact, the passage of the Urban Mass Transportation Assistance Act of 1970 provides financial aid to local communities to meet urban mass transportation requirements. This program, begun in fiscal 1971, provides for 3.1 billion dollars for the following five years.

since an adequate level of service should be provided at all times. However, due to the variations of passenger demands during different periods, transit service frequency and the associated fleet size should be determined flexibly in response to these variations.

The derivation of total cost-utility, a measure of transit system performance, can be an important basis for the determination of public subsidy since the amount of subsidy may well be justified due to the cost savings derived by the transit model.

Consequently, it is reemphasized that, in planning an optimal transit system, there is a definite need for a validated tool which will provide reliable alternatives to current bus transit service configurations.

The Concept of Systems Analysis in Planning for Bus Transit

In planning for bus transit, the planner must choose among a set of alternative systems of bus routes, headways, fleet sizes and bus stops. The optimal transit system is determined based on the total cost comprising of bus operating and ownership costs, passenger cost and bus fares. This problem of finding the optimum transit system is particularly critical since a sub-optimum system causes extra cost to the operator as well as the passenger.

In order to produce an optimum bus system, a multitude of interacting variables must be considered. It is also important that a bus route inside a system be viewed as a part of the system rather than as an isolated one. In the past, only the costs and benefits directly associated with the route being analyzed have been considered. However, an improvement of one bus route or addition of a new route in the system can result in benefits in other parts of the system. This is referred to as "system effect", which will be analyzed by the model developed in this study.

The system effects, therefore, must be measured by the performance of bus transit service in view of the overall system objective, which reflects the essential elements of the system.

The quantifiable system measures generally consist of accessibility of service, waiting and traveling time, passenger service time, delays due to traffic congestion and signals, bus operating costs and the ownership cost of the bus fleet.

Different system measures for alternative bus systems usually arise from variations of such bus transit system elements as route structure, service frequency, fleet size and service mode. In complex bus systems in large cities, the variations of the above elements are almost infinite and there is a need for planning tools which can determine the optimum bus system configurations among alternatives through systematic investigations. This consideration necessitates the application of the concept of systems analysis to the study of bus system operations and planning.

The central aim of systems analysis is the development of mathematical models that permit a formalizing of the problem under investigation in precise mathematical terms. For the study of bus transit operation and planning, an emphasis is made, in this respect, on the application of the systems techniques of linear programming and dynamic programming.

Besides these techniques, a variety of other techniques have been developed for a wide range of systems application. Among these, such techniques as game theory, queueing theory, inventory theory and simulation also have been successfully applied to various aspects of the systems problem.

The revolution of computer technology, in addition to the rapid advancements of system analysis tools, has given great impetus to applications of systems analysis in a variety of contexts in the field of transit planning.

Study Objectives

The purpose of this study is to develop a bus transit model capable of establishing an explicit relationship among the major factors of the bus transit system - transit users, transit operators and transit system, to compute the bus transit figure of cost-utility, the measure of system performance.

The use of formal planning tools in the analysis of a bus transit system has been directed primarily toward the costs and benefits associated with particular transit routes isolated from the total system. The reasons for this isolated approach are partly because the number of transit factors must be limited to make an operational model, and partly because direct impacts from particular bus routes tend to draw more attention than the complex transit system effects. The result is that the analysis is not truly system oriented, but piecemeal and localized.

Here, the emphasis is to incorporate the essential elements of bus transit system into the transit model. The analysis and the derivation of an optimal transit system, then, are made using versatile systems analysis tools and efficient computer programs. The use of modern computer technology with well organized systems tools enables the investigation of bus transit system effects as well as economical consideration of many relevant transit factors.

The study addresses itself directly to the question of whether or not a newly proposed bus route can be extended to the existing bus transit system to bring about reductions in the total cost measure, and if it can, what will be the optimum level of service to be introduced to the system. The answer to this question is important for the public transportation planner to determine a program of transit improvements to be included in the coordinated transportation plan.

Furthermore, an analysis of the existing system using an analytical planning tool will be helpful in determining the extension or curtailment of service, and the effective coordination of bus transit with other forms of transport in urban areas.

The study is designed mainly for the need for systemwide transit planning technology. The study does not include the development of specific vehicle schedules or manpower assignments.

Within the framework of bus transit planning, the study has two specific objectives: One is to develop a two phase model to evaluate bus transit operations and to plan systems improvements. The other is to apply the model to explore the feasibility of cost savings for the proposed bus system using automatic computational routines especially developed for this purpose.

Approach Toward Bus Transit Modeling

This study attempts to improve bus transit service by optimizing the systemwide configurations of bus route, service frequency and bus fleet size which are operational in nature.

The approach of the study towards this goal is characterized by the use of mathematical programming techniques. The programming techniques utilized for the purpose of formulating the model are first a linear programming algorithm and second, the dynamic programming process. The former investigates the transit system operation during a specific schedule period which has fixed system characteristics as to the route network, service frequency, fleet size and passenger demand profile. The latter determines the optimum size of transit improvements to have an overall system effectiveness throughout all schedule periods. The term schedule period refers to a partition of time to represent homogenous travel characteristics of a day and a week.

The bus transit model is, therefore, a joint model consisting of the first-phase, linear programming model and the second-phase dynamic programming model. These two phases of the model are interrelated with each other. For example, the output of the linear programming model for the optimum transit operation becomes an input to the dynamic programming model to make a decision on the planning of the optimum system improvements during all schedule periods.

In determining where, when and how to alter the transit system variables, the approach taken is defined as follows:

Given:

1. Passenger demands for bus transit service between major traffic generators.
2. Street network and existing transit routes.
3. Service frequency of all existing routes representing passenger carrying capacity of each link of the route network.

4. Transit demand elasticity over service. This is expressed as a linear approximation of the relationship between the load factor and the number of operating buses.
5. Transit fleet size of all existing routes.
6. Operating budget of transit service for the chosen study network.
7. Passenger Origin-Destination and distribution over time.
8. Properties of schedule period such as duration and demand density.
9. Physical traffic characteristics of the study network such as street capacity and bus stop locations.
10. Cost parameters for transit operating expense and passenger time.
11. Bus fleet ownership cost.
12. Transit planning policy on how different cost components should be weighted.

Determine:

1. System benefits of adding or deleting a bus route.
2. Service frequency to operate on the new proposed route.
3. Bus fleet size to provide optimum service during different service periods.
4. Bus and passenger flows during different periods to provide optimum transit operation.

5. Cost-utility of transit operation such as operating cost, passenger revenue and passenger cost.
6. Incremental costs due to the change of network characteristics such as operating speed.
7. Incremental costs caused by the change of transit service such as headway and fleet size.
8. Effects of bus ownership costs on transit system configuration.
9. Impact of transit parameter variations on transit cost and service preformance.

Solution:

The two-phase transit model developed in this research is used to compute incremental costs of extending new routes, abandoning routes or modifying the service frequency, fleet size and other attributes of transit systemwide configuration, i.e. link operating speed, delay, demand and cost factors. The model formulated jointly in the linear and dynamic programming problem is intended to solve both the fixed transit operation during one period and the dynamic planning over the entire transit service cycle.

Synopsis

Chapter II offers a discussion on the conceptual framework for the development of the bus transit planning model. General concepts, strategy and new transit planning techniques are discussed as utilized in the study. This section of the thesis also outlines and describes the transit system objectives, components and major system elements of the two-phase joint transit operation and planning model.

Chapter III extends the discussions on the structural elements of transit services to formulate the analytical relationships between transit performance and system variables. This chapter discusses the analysis and selection of major components of transit service environments as related to the model.

Chapter IV develops the first-phase transit operations model and identifies, relates and specifies the interrelationships of system elements. It develops the formulation of the transit operations problem into a linear programming problem specifying an objective function and various system constraints in mathematical terms.

Chapter V develops the second-phase transit planning model and extends the single period transit operation to multiple schedule periods for transit planning. It develops the dynamic programming process of the transit planning problem. It also presents criteria for the evaluation and design of the structural elements of the model.

Chapter VI presents an application of the transit model in a case study format to illustrate the capabilities of the transit model through its application and an evaluation of results.

Chapter VII offers a summary of the findings and an appraisal of the model with regard to its limitations and the implications of the results to the study objectives. It also presents suggestions for future research needs.

In the Appendix, a selected review of the literature of transit operations and planning analysis is presented. In another section of the Appendix, flow charts, listings of computer programs and graphical supplements to the text are also included. In addition, the computer inputs and outputs for the case study conducted using the transit model are also attached in the last section.

CHAPTER II

A FRAMEWORK FOR THE DEVELOPMENT OF THE BUS TRANSIT SYSTEM MODEL

Introduction

In this Chapter, a general analytical framework for bus transit simulation is described. From this framework, a study of transit service impacts on the urban community is developed. The Chapter is also devoted to the development of the theoretical background that permits a formalizing of the transit operation and planning problems into analytical relations.

The Concept of Modeling as a Tool for Bus Transit Studies

The use of models in transit planning analysis is as much a philosophy for approaching a complex urban problem as it is a technique. A model is a symbolic representation of a real world system. The function of a transit planning model is to establish a logical framework within which the relationship between the variables and parameters of a transit planning problem can be specified for the analysis of the overall system. The urban transit study is concerned with determining the implications of future policy decisions upon urban transit systems. Later, the model is applied as a guide for policy in the operations and planning for a bus transit system in a specified study area.

Essentially, the variations in transit policy constitute different

transit service conditions, i.e. different headway, fleet size and route configurations. The model generates a measure of system performance, cost-utility, by testing and analyzing the extent of both service and user requirements for the transit system.
The measure of system performance thus generated can comprise a basis for transit policies on system operation and planning functions.

The modeling concept posed in this study dwells on four premises.[1] First, a model should be a product of a logically consistent organizing concept. Its design should be based on some theoretical framework to represent the process of transit systems as it occurs in the real world and to focus on the transit operation as it actually takes place within the urban transportation network.

The second premise is that it should have a function which relates both short term and long term transit operations in a continuing process. The function should suggest long range transit policy with built-in features for adjustment and modification. Accordingly, the model should be designed to take account of major transit system variables as well as parameters that transit planners consider in selecting transit operation and planning policies.

The third premise is that the model should have dynamic characteristics so that the evolutionary nature of transit service improvements can be analyzed. For example, the service improve-

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[1]For specific criteria for model design, see (31) P. 102.

ment at one point in space and time may influence another point at some other part of the system. More specifically, modification of fleet size along one route during the morning rush hour may affect another route during the off-peak period. Ideally, a transit model should be able to analyze the need for transit service from an individual point of view, rather than from the "mass" point of view. For example, if a bus route is designed purely based on area coverage or demand density, it may overlook individual trip characteristics as to user access, path and physical properties of street uses. Also, due to the magnitude of the investigation and the computations involved in transit planning, the decomposition of a large problem into smaller planning entities should be introduced. The decomposition sometimes requires an investigation of dynamic relationships between smaller planning entities to yield a realistic analysis of the whole system.

Lastly, the model should have the adaptability to high speed computer technology because transit operations and planning in an urban community are very complex and cannot be analyzed in simple abstract forms. The modern computer system with its capability of efficient data handling and storage can be utilized for the investigation of urban transit operations and planning at tremendous savings of time and cost.

A Conceptual Development of the New Technique for Transit Planning

The conceptual framework for determining an optimal transit system operation is based on transit operational characteristics which are identified from the observation and analysis of actual transit systems. As a first characteristic of a transit system in an urban community, the fixed nature of the transit route configurations is identified. A bus route is designed and implemented to serve a specific passenger demand in such a way that reasonably direct connections between major urban activity centers can be provided.

However, once a bus route is established, then it remains fixed to serve anticipated passenger demand until a major change of demand absolutely necessitates the modification of the route structure. Often the routes remain fixed regardless of the shift of demand and other variations of bus transit service conditions.

This seemingly detrimental aspect of transit service has its own virtue too, in the sense that it provides consistent service which will help the potential transit users to avoid confusion arising by ever-changing bus routes without proper advance notifications. On the other hand, it is also true that this fixed route character of transit systems reduces the operational efficiency and sensitive response to the changing pattern of passenger demand.

Meanwhile, bus transit has the flexibility and adaptability to meet the changing service requirements in contrast to fixed route structure of rail transit. However, bus transit routes should be investigated during planning stages well before the implementation of actual service in order to utilize the inherent flexibility unless a complete demand responsive system is established with an instant real-time communication system between the user and the transit operator.

In order to make the best use of flexibility, it is imperative to have an optimal selection of route location during planning stages, using such a model as proposed in this thesis to meet all stochastic demands during all planning periods over the entire range of service areas.

In connection with bus routes, it is also observed that scheduling of bus service on transit routes on a continuous time scale has a distinct character of cycling. A cycle is a repetitive function of phenomenon or process. As an obvious example of a cycle, traffic signal cycle is illustrated here.[2] It has a constant cycle length and uniform splits such as green, amber and red to assign right-of-way to the different approaches of an intersection alternately. Once a cycle is selected, then any length of time can be serviced by continuing cycle and splits.

Likewise, any planning period of transit service can be defined by using the concept of a cycle in terms of bus use and service provision. Observation of a bus timetable easily reveals

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[2]For further discussion and design of traffic signal cycle, see (5).

a period of a week for the transit service cycle.[3] This cycle includes all distinguishable service and demand characteristics in the weekly cycle such as (1) weekday evening peak period, (2) morning peak period, (3) weekday off-peak period, (4) Saturday peak and (5) off-peak period, and (6) Sunday period.

Consequently, this study identifies the transit cycle and suggests its use as an entity of transit planning.

Another interesting system characteristic which extends from the concept of transit schedule-cycles is the partitioning of the weekly cycle into schedule periods. This partitioning enables the use of a multi-stage decision process[4] to determine transit policies for each individual schedule period for the system. Specifically, the process develops service frequency and fleet size required for the optimal system.

A decision at one schedule period influences a decision at another period, as the optimal solution for one period may not be the best for another period. Subsequently, a systematic approach should be applied to determine the policy at each period in order to produce overall system effectiveness.

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[3]The scheduled bus distributions over time was examined based on November, 1971 bus block diagrams of line No. 25-26 in Newark, New Jersey.

[4]See Nemhauser (62).

Furthermore, one characteristic of the transit system which is used as a building block of the model is also derived from the realization that transit system variables have different degrees of freedom for modification and alteration.[5] Since transit service environments keep changing due to the shift of population and change of land use patterns, the transit system should be able to incorporate these changing processes to meet the varying service requirements more efficiently. This can be best accomplished by modifying system variables according to their degree of freedom.

Accordingly, before any system modifications are implemented, the proper order of major system variables should be identified with regard to their degrees of freedom and ease of modification. As observed in actual transit operation, the degrees of freedom are realized in the descending order of service frequency, fleet size and lastly, transit route configuration.

The reasoning behind these orders of freedom is easily seen by inspecting the operation of bus transit. For example, bus service frequency, the headway provided by a bus fleet, can be easily adjusted within the range of potential service frequencies. This is so because the service frequency of a given bus fleet may have an unused portion which can be utilized to expand and modify the service frequency. This is especially true when a

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[5]For guides in developing transit improvements, see (105) "Recommended Standards, Warrants, and Objectives for Transit Services and Facilities."

given bus fleet produces a maximum capacity during peak rush hours, say weekday morning and evening peak hours, while it uses only a portion of that maximum capacity during off-peak period, say Sunday. If additional demands require more service on Sunday, then the unused part of service should be first utilized before the fleet size is increased.

The same reasoning can be applied to the transit route. Once a transit route is installed on a street network, an adequate bus headway is provided by a bus fleet to realize the demands along the route. However, the passenger demand pattern can be shifted and a change of system may be required. In this case, a change of system in response to the change of demand profile should be first realized through the modification of service and associated fleet size.

With the understanding of major system variables of headway, fleet size and route configurations, determination of an optimal transit system operation is carried out by computing cost-utilities incurred in providing the existing and proposed bus transit service.

The actual value of the cost-utility of a transit service is calculated based on cost and performance actually experienced by the transit operator as well as the transit users. Also, transit network characteristics are considered in the derivation of cost-utility figures of merit since they contribute to the system measure directly in terms of service quality. If an independent value of cost-utility is established for each combination of demand pattern, service level, and physical network configuration, it can be a useful measure for transit planners to compare different demand-service-system alternatives to choose an optimum solution.

In fact, the number of above transit system combinations is tremendously large. Therefore, it is recognized that a bus transit model that would systematically determine the feasibility of a new route and the cost-utility of different system configurations would ultimately prove beneficial to bus transit planners, who need analytical tools to evaluate transit systems.

Bus Transit System Parameters and Variables

After the conceptual framework defines the basic structure for the transit model, the analytical design of the model is undertaken by first investigating the variables and system parameters which affect the quantified study objectives.

The parameters considered in the structural analysis are those that are descriptive of the performance and operational characteristics of the transit system. As major parameters for the first phase, transit operations model, the transit patronage, route network configurations, operating cost, travel time, load factors, passenger time value and the passenger revenue are selected.[6]

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[6]See Lisco (98).

For the second phase of the transit planning model, potential ranges and increments for service frequency and fleet size, and the unit bus runs of each schedule period are chosen as parameters. In addition to this, annual bus ownership cost and the transit usage weighting factors are considered.[7]

As decision variables which influence the outcome of the transit system, the amount of transit service provided and the transit demand realized are identified. The computational routine is such that the above variables are computed in an optimal manner. In more detail, the transit service provided is further specified in terms of service frequency and the transit fleet size. The demand realized refers to specific information on passenger flows and their service characteristics such as travel path, load factor and passenger time costs.

The detailed definition and relationships of the above system parameters and variables are further discussed in Chapter IV and V including the development of a set of system equations.

Transit System Goals and Evaluation Criteria

The transit system objective as proposed in this study is to optimize the objective function which is an explicit mathematical statement of transit service output. This quantitative measure of

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[7]For regression relationship between operating expense and service, see (71).

system operation and performance is highly useful in the determination of optimum transit operating and planning policies.

In practice, the goal of the bus transit system can vary widely, ranging from the minimization of operating cost to the maximization of profit or other combined social goals such as ridership with a certain percent of profit. An array of objective functions most frequently investigated by the transit operator consists of either maximization or minimization of certain properties of the transit system output subject to a set of transit service constraints. For maximization, such properties as transit profit, revenue and ridership are usually considered, while for minimization, operating cost, fleet size or manpower requirements are investigated.[8] These objective functions can be used singularly or in combination. For combined objectives, two or more single objectives are related and investigated concurrently to represent the system performance in a more realistic way.

This study deals with a wide variety of system objectives which are importantly related to the major transit system components, that is, the transit user, operator and the system.

For this study, four major elements are selected to formulate the objective function of the transit model. They are passenger

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[8]For further discussion of manpower assignments for bus transit, see Elias (81).

cost, bus operating cost, the passenger revenue and the vehicle ownership cost.

The first, passenger cost, are those costs which are seldom considered quantitatively by transit planners. These costs occur to passengers during the use of the transit system in terms of time spent for service. For example, time spent for walking, transferring and riding are considered as important passenger costs. The second, transit operating cost, refers to the cost incurred to the transit operator in terms of system products such as bus-miles and bus-hours. For example, the bus operation for an hour or a mile requires expenses like wage, fuel and tires.[9] The third, passenger revenue, is the potential income derived from the collection of fares.[10] The fourth, ownership cost of revenue vehicles, is the cost incurred by bus vehicles which are introduced into the transit system. This cost is dependent upon the size of bus fleet retained for a specific level of service which causes costs to the transit operator in terms of purchase, and other financial fees.

Based on this ownership cost and the number of buses that should be introduced in the system during a particular period for an

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[9]For correlation matrix of bus cost parameters, see (71).

[10]For fares of Public Service lines in Newark, New Jersey, see (106).

overall optimal transit operation, the transit planner may consider an alternative to the outright purchase of bus vehicles. By leasing vehicles for peak period use rather than owning them, the total bus ownership cost can be reduced because of more effective equipment utilization. A further discussion of ownership cost can be found in Chapter V.

The above mentioned cost elements are combined as a criterion for evaluating alternative transit system configurations. The first three elements - passenger cost, transit operating cost and passenger revenue are incorporated into the first phase transit operation model. The difference between the passenger revenue and transit operating cost is the transit operating profit that does not account for vehicular ownership costs, which are considered in the second phase transit planning model. Therefore, the overall optimal solution provided by the model would be based on the quantified total cost of the objective function that takes account of all major items of transit performance.

In summary, the bus transit evaluation criteria proposed in this study are unique in the sense that they integrate all major transit system components, i.e. the transit user, operator and the system. Traditionary, only those costs related to the transit operator and network have been considered, overlooking inconvenience and delay incurred to the user. Therefore, the new concept of the evaluation criteria may be useful for a transit planning agency at the state level where policy-making is done on the basis of overall system effectiveness.

Bus Transit Operation and Planning

As discussed in previous sections, the study consists of two distinct phases of work to develop an analytical tool of wide application within the framework of fixed route bus transit. It is appropriate at this point to consider the objective of each phase and their relationship with the overall mechanics of the model.

Transit service within urban areas is characterized by the fixed nature of their service and route network during schedule periods. However, the stochastic character of urban travel requirements makes it necessary to have a certain variation of service to meet the prevalent demand pattern in a more efficient way.

Accordingly, the first phase of analysis concerns bus transit operation during one period, for example, weekday morning peak period. This period has a known patronage which is served by a fixed number of buses with a constant headway. The objective of this phase is to have bus transit operation in such a way that the total system performance measure would be optimized.

The second phase of the study combines each of the first phase transit operations for a given period with all others so that transit service can be modeled on a continual basis. In fact, there can be many different ways to combine transit operation for entire planning periods. Consequently, by aggregating bus operation of each period for the entire weekly cycle in an optimal way, the second phase can provide a dynamic response to the varying nature of transit demand and trip characteristics.

Therefore, the effort toward the development of the two phase model centers on two system characters, that is, fixed character of bus transit operation during a single schedule period and the dynamic nature of the transit planning for the provision of continuous service.

Multi-Stage Decision Approach Toward Bus System Planning

In planning continuous transit service, the cyclic pattern of demands is recognized for a design of a basic time unit of planning. For example, an observation of existing transit schedule and passenger demands indicates that a period of one week, which includes regular weekday and weekend, usually includes all different characteristics of transit service environment. In addition to this, monthly and seasonal variation can be added as a useful incentive for system modification. However, the usually negligible change in month and season simplifies the selection of the transit planning cycle to be a weekly period.

Subsequently, this study deals with a weekly period as a unit for analysis and planning of the transit system. Therefore, once the transit operating and planning policies are determined, the service can be provided continuously with a cycle of a week.

A cycle of a week is further partitioned into weekday evening peak period, morning peak period and off-peak period, etc., to represent the homogenous trip characteristics within the cycle. The subdivided interval is referred to as a schedule period. Once a basic planning cycle is partitioned, then the nature of the multi-stage decision process can be utilized to determine service frequency and fleet size for each schedule period.

A multi-stage decision process is a technique to make a sequence of interrelated decisions to have overall effectiveness of decisions. This process is characterized by the fact that the overall decision problem can be divided up into stages and each stage requires a policy decision to yield maximum system return.

This nature of multi-stage decision process is captured in combining the transit operation of each period as well as optimum decisions in the same period. The decision refers to the service frequency and fleet size for the overall optimum system configuration.

Service Mode and Bus Stop Organization

Once the bus route under investigation is located and the optimum service frequency is determined, details of operational problems should be considered.

One important problem, in this regard, is to locate bus stops along the radial bus route which carries downtown oriented commuter type passengers with high directional variation. An explicit mathematical statement concerning bus stop location is very useful, if it can be developed, in measuring quantitatively the efficiency of transit performance which is related to operational delays at bus stops.

The radial bus route carrying commuters to and from the downtown area requires a fast inbound and outbound service during morning and evening rush hours to satisfy demands with high peaking characteristics. The frequent stops at series of bus stops along the route incur unnecessary delays to the through passengers.

The demands for bus service are distributed over bus stops and different periods of the day. During peak periods, it may be feasible to employ two modes of service which are express and local in order to reduce unnecessary intermediate stops of the downtown oriented through passengers. The provision of two-mode service makes it necessary to group a series of stops into the express and local stops.

More specifically, during peak periods, a selection of bus stops can be designated as express stops and the passengers at these stops may be served by both express and local, and the rest of the stops by only local buses. If a local bus is dispatched, the bus makes a stop at all stops and if the express is picked, the bus stops only at express stops traveling non-stop at local stops. The bus stops organized in this way would meet passenger demands in a way to favor major downtown oriented passengers. As a result, the total passenger delay on the bus route will be reduced.

Transit data usually available for the purpose of organizing bus stops include bus stops and their locations, downtown oriented demands, number of passengers boarding and alighting at each stop, average operating speed and vehicle performance characteristics such as acceleration and deceleration. Once a bus route is located using the transit model, then bus stops can be grouped into local and express by comparing total delays incurred by different configurations of bus stops.

Strategy for the Development of the Transit Model

In implementing the concepts discussed in previous sections, the sequence of analytical steps required for the development of an operational model comprises the design of a two phase joint mathematical programming model. One phase is for transit operation and the other phase for transit planning.

This type of investigation may have two distinct approaches.[11] The first approach is a macro-analysis which is characterized by progressive disaggregation of complex relationships into mathematical expressions to estimate the system performance. The second approach may be described as a micro-analysis. This approach first defines the relationship of the subsystem and then combines them in a progressive aggregation to yield system evaluation measures.

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[11] See Lowry (32) P. 160.

Both approaches have advantages and pitfalls. For example, the macro-model approach has the advantage of concentrating on those relationships contributing directly to the objective, thus simplifying the overall formulation and data requirements. One shortcoming is that it does not guarantee the causal relationship between functions. By contrast, the micro-model has the advantage of having well defined and accurate relationships, yet the micro-model requires the investigation of variables which may not affect objectives, and thereby demands much more data.

The study is, in essence, a macro-analytic approach to the development of a two phase model for transit system operation and planning. This approach resulted after reviewing the features of both the macroscopic and microscopic approaches, the requirement of a predictive capability for the model, the data requirements and the use of available analytical tools.

CHAPTER III

ANALYSIS OF BUS TRANSIT SYSTEMS

Introduction

In this chapter the basic structural elements of bus transit systems are investigated in order to develop an analytical framework for model building. This process of forming a logical basis consists of an identification of the transit system components by first defining (1) the transit system, (2) the operator, and (3) the user.

Transit System Components

For an analysis of a bus transit system, the transit service area should be defined geographically in sufficient detail. In defining the service area, first the segment of an urban region is selected. Second, the street network within the segment is further identified to show existing bus transit routes and to build proposed routes.

A Bus Transit Corridor. As an example of a transit corridor, the segment of an urban region is termed a corridor when it includes radial roadways connecting a downtown area with major activity centers. The connection is made through major streets which efficiently move auto and passenger traffic. This corridor usually includes a radial roadway, a major street which is characterized by relatively wide pavement, uniform traffic control devices, and roadside facilities such as curbs or guard rails to separate auto traffic from pedestrian traffic. In addition, major streets are distinguished most conspicuously by favorable signal progression for predominant traffic flows.

The corridor analysis is specifically designed to identify and analyze a study area whose bus transit operation is independent of any other corridor and whose trip characteristics are relatively homogenous. The hypothesis on independence and the homogenity of trip character is realistic and practical due to geographic separation of transit route area and the limitation of walking distance.[1]

In other words, a corridor defined by a geographic barrier or maximum walking distance can be an entity or unit for the analysis and design of bus transit system operation. This concept indicates that the change of a system configuration such as either route or service frequency or both within a single corridor does not affect the bus operation in any other corridor.

The division of a large area with non-uniform traffic characteristics into corridors helps to reduce the size of the problem under investigation. Thus, the use of the corridor concept makes the analysis of a transit system feasible and managable.

Consequently, a large city is divided into a set of corridors. Each corridor is to include at least one major radial arterial

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[1]For a discussion of limitation of walking distance, see Peterson (36).

street connecting the Central Business District with outlying areas. The corridor boundaries should be chosen with consideration for the present bus route configurations and topography. The boundaries should be placed so as to contain at least one radial route and to cross a minimum number of radial bus lines. The study area and a typical design of corridors are shown in Figure 1.

In a transit study, the importance of the radial route remains critical because passengers are concentrated on this line and competitiveness of bus transit in a large city is most favorable to radial movements due to high density of population, easy access to the bus service and relatively high bus operating speed along the radial route.

Street Network. A street network within a corridor consists of many features in order to move people and goods efficiently.

The street network of the study area is shown in Figure 2. This network includes the existing bus routes and those street links which can be used for a bus route in the future. Generally, the number of existing streets qualified to be a potential bus route is limited due to street approach width, turning radii, parking conditions and existing traffic volume. Since streets whose geometrics are not adequate for a bus route can be taken out of the study route network, the skeleton network under investigation consists of only adequate streets for a bus route.[2] Subsequently,

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[2]For geometrics of bus runways, see (86) and (65).

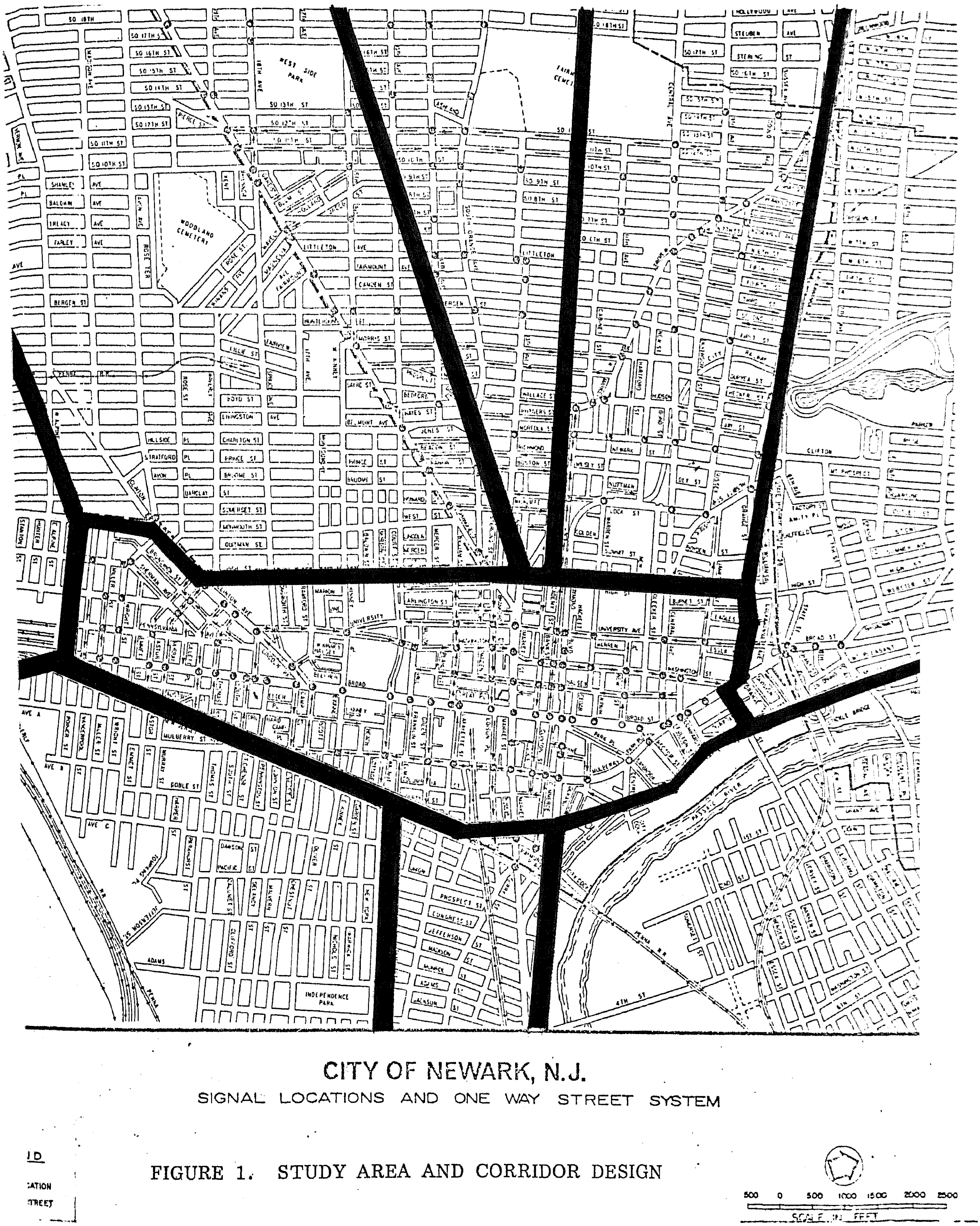

FIGURE 1. STUDY AREA AND CORRIDOR DESIGN

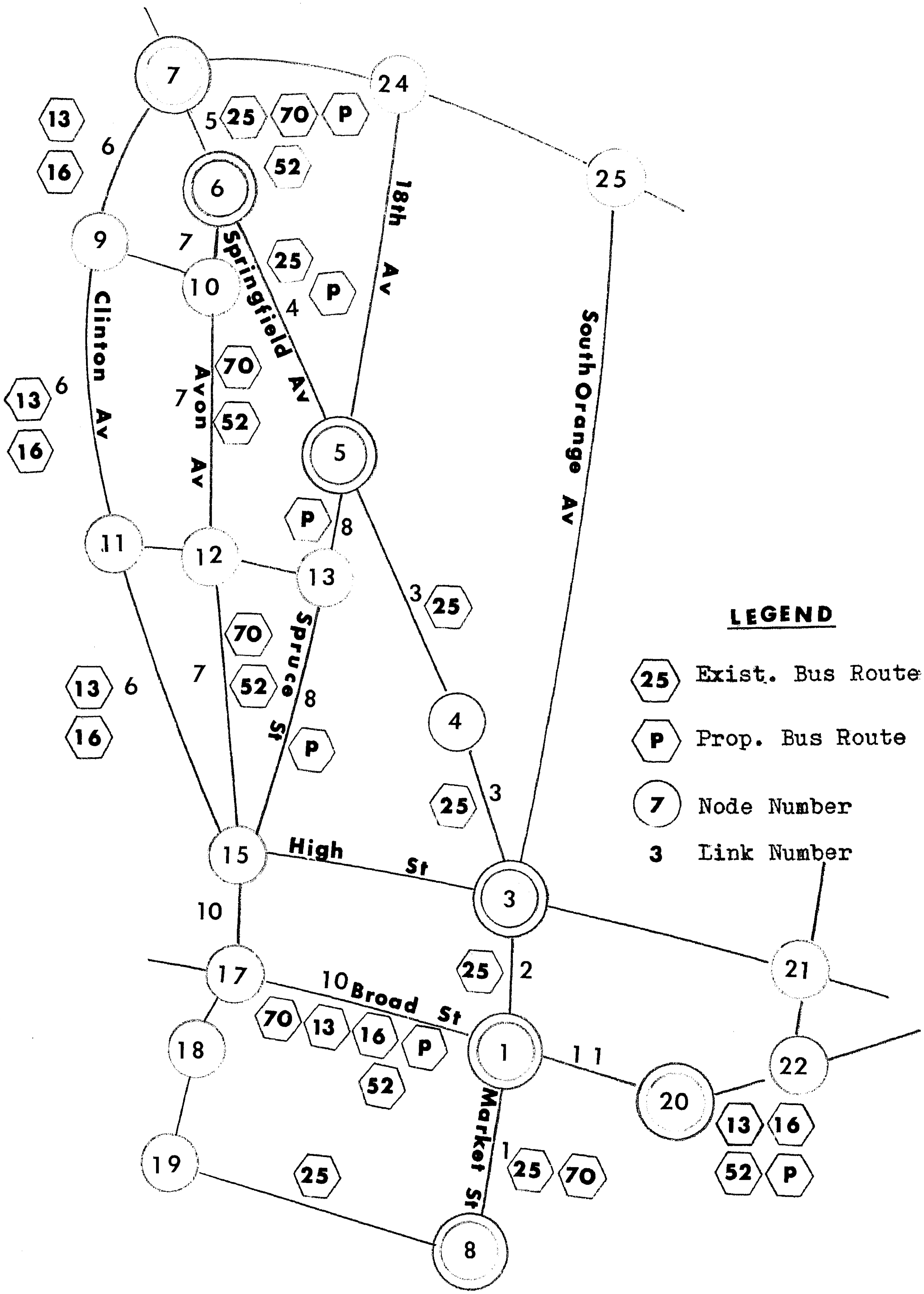

FIGURE 2 STREET NETWORK IN SPRINGFIELD CORRIDOR

data is collected only for this skeleton network whose size may be much smaller than the original one.

For the purpose of representing transit service in mathematical terms, the network is first defined by means of links, nodes and chains. A street link is a segment of street connecting two nodes with possibly street intersections on both ends. It is characterized by uniform link properties pertinent to link travel time, link operating cost and passenger carrying capacity.

A node is an intersection of links and can serve as a point of passenger demand. Usually a passenger demand is the trip need between two nodes in the network during a schedule period. In fact, an origin or a destination can be any point other than a node if the connection between the point and the node in the network is defined. For a city with a high density of street network, a node can be satisfactorily used to represent any demand since any point in the area is close to a node of some sort.

A chain is defined as a sequence of links to go from an origin to a destination. A chain, therefore, consists of a set of links connecting any two nodes consecutively. Since most nodes in the network are connected by more than one link, there can be more than one chain to connect two nodes. In fact, if all possible chains are considered, then there can be too many chains for investigation. However, chains which are reasonably direct can be easily determined by observation.

When a passenger travels by bus, he is interested in his total travel time. This travel time consists of not only riding time along bus routes but also times required for walking, waiting and transferring. In fact, the last three time elements have very important bearing on the success of transit service. Consequently, a set of imaginary links representing walking and transfering are utilized in this study to trace the actual path of travel and to compare travel times by alternate routes.

Once a network is defined, the passenger demands between any two Origin and Destination nodes can be realized by the flow of passengers on chains connecting corresponding nodes.

Building the Proposed Bus Route. A representation of both existing and proposed bus route networks is essential for the bus study since transit networks directly affect transit users and operators. In fact, the actual configuration of the route network is the most influential system component that characterizes transit service environments.

The detailed route description of the existing bus operation can be made based on bus route maps, schedules and run guides. However, the design of a proposed bus route which is evaluated as an alternate modification of the existing transit system, should be appraised based on not only quantitative transit system criteria such as total cost-utility of service, but also qualitative criteria[3] such as simplicity of bus route and avoidence of a long loop. This is partly because bus routing should have desirable characteristics recognized as a qualitative routing standards and partly because an evaluation of the proposed route based on routing standards would help to reduce possible system alternatives for investigation.

In designing a bus transit route, emphasis should be first given to qualitative criteria for upgrading transit service quality in terms of:

1. Passenger satisfaction and convenience.
2. Minimization of required transfer between various bus lines.
3. The improvement of operating speeds and reduction of delays.
4. Provision of reasonably direct, non-duplicated and simple routes.

Operator Components

For the development of transit improvements on prescribed bus routes, the operator must consider such relevant components as service frequency, fleet size and operating budget. These components are interrelated among one another and require certain

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[3]For standards for routing, see (105).

service conditions.

Bus Service Frequency. Service frequency is a measure of amount of service given on the transit route network. It is also expressed as headway which is the time interval between bus vehicles. The amount of service provided must be given careful attention since it is closely related to the financial outcome of transit service. For example, the product of transit service is potential bus riders which exist only during the time of service, so the unused part of the service becomes a waste of equipment and manpower.

Consequently, the frequency of service provided should be evaluated and controlled on a continual basis. The purpose of the evaluation is to minimize waste and operating costs involving the high wage rate,[4] material cost and maintenance fees required for the provision of transit service during each schedule period.

The existing service frequency on each link of bus network is derived from the bus route map and block diagrams. When bus lines running on each link and their frequencies are known, then the total service frequency on a specific link is computed by summing up all related service frequencies.

In addition to existing service frequency, new services provided

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[4]For discussion of historical yearly increase of hourly wage rate, see (68) Charts 1 and 2.

on the proposed bus route should be considered for the analysis of the feasibility of the new route. The new service is added along the proposed route by means of a constant increment so that the amount of service can be adjusted for different headway configurations. The actual numerical value of an increment can be varied depending upon the required accuracy.

Once service frequency is quantified for links in the network, the passenger carrying capacity of same links can be computed based on the frequency, average bus occupancy rate and bus capacity. The occupancy rates are affected by the time of service, i.e., peak or non-peak periods. The occupancy rate is empirically derived for the transit model from the existing bus data and it is termed a load factor. Numerically, a load factor is the number of bus riders per bus during a specific schedule periods.

The number of bus passengers for a specific Origin - Destination pair during a particular period is seemingly fixed. However, it is observed in reality that the demand itself has an elasticity over service. In other words, demand responds to the amount of service provided.

The reasoning behind this is that as more service is provided, the better the level of transit service becomes, which in turn will induce more people to switch to the bus transit system from other modes. Yet, the rate of increase may become smaller when service surpasses a certain limit. This is because the total trip demand generated in a given area is relatively constant and it restricts the diversion of trips from other modes, i.e. passenger cars to bus transit. Typical load factors are approximated linearly as a function of either bus passengers or bus frequency is shown in Figure 3. The derivation of the limits of passenger flows with higher load factor requires a large amount of data collection and analysis of demand elasticity.[5] The demand elasticity should be analyzed on a long range basis for different trip purposes and trip makers.

Bus Fleet Size. The transit demand during a particular period governs the choice of bus service frequency. This, in turn, determines the minimum bus fleet size to be retained for the service. Nevertheless, the fleet size required for overall transit service during an entire planning cycle can not be determined based only on the service frequency required for one specific schedule period, say, weekday peak period. This is because other periods may need different fleet sizes for the overall optimal transit operation.

Accordingly it is useful to compute fleet size required for each individual schedule period and then to derive one overall optimum fleet size. For this purpose, the schedule periods are arranged in

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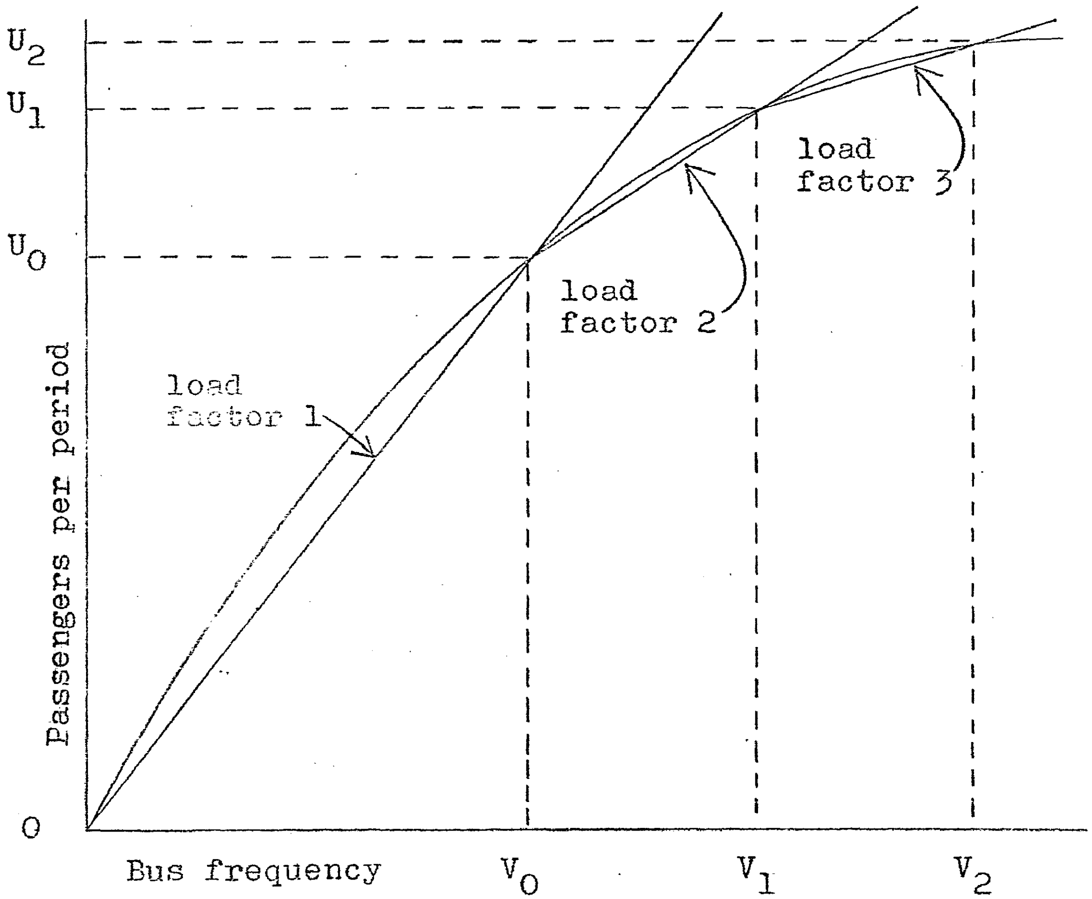
[5]Demand elasticity is defined as the change of demand rate due to the change of service. For further discussion, see Hartgen (89).



Load factor L_1 for passenger $\leq U_0$, Bus $\leq V_0$

Load factor L_2 for $U_1 \geq$ passenger $> U_0$, $V_1 \geq$ Bus $> V_0$

FIGURE 3 PIECE-WISE LINEAR APPROXIMATION OF SERVICE ELASTICITY the order of demand densities[6] of schedule periods so that fleet size for smaller demand is first computed and then next higher demand. This arrangement of schedule periods facilitates the determination of different fleet sizes, that is, fleet size utilized for the entire planning cycle, the fleet size required for individual periods and the fleet size required only during the highest peak period. The first type of fleet is a base fleet while the other two are non-base fleets.

Once this information on different types of fleet sizes is determined, then actual provision of bus vehicles can be arranged by selecting types of ownership, i.e. publically owned, privately owned or rented vehicles. The percent utilization of a bus fleet during a planning cycle can also be determined based on total periods of usage. A determination of the number of bus vehicles required during each schedule period would facilitate the development of factors to weight bus fleet ownership cost. The weighting of the ownership cost would be based on the annual bus ownership cost, total operating hours and the period during which the bus fleet should be incremented to meet the demand.

Schedule Period Operating Budget. An operating budget of transit service is to ensure that required expenses for the provision of service should be within a predetermined budget limit for a specific schedule period. The ever increasing cost of wages,

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[6]Demand density refers to number of passengers per unit of time.

maintenance costs, material costs and taxes required for transit service call for an efficient control over expenses by transit management during each schedule period.

In order to impose a budget limit of operating cost, an actual bus operating cost incurred during each period has to be computed. The bus operating cost is computed based on bus-hour or bus-mile. Since the operating cost occurs due to direct wages, fuel, tires, repairs and service, the bus service output per bus-hour or bus-mile is well correlated with the operating cost. In fact a previous bus transit cost study[7] shows a high correlation between operating cost and bus-hours with coefficients of correlation ranging from 0.91 to 0.99 depending upon the catagory of fleet sizes.

The available bus transit data is usually grouped according to salient features of transit system configuration such as fleet size, service area and ownership status. This grouping would help to make a statistical analysis to derive a set of linear regression equations of bus operating cost for different transit service conditions.

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[7]See (71). Transit data from the American Transit Association was analyzed for correlations among bus parameters to identify significant variables for bus operating cost. The correlation between operating cost and bus-hours was found to be statistically significant for bus fleets stratified as under 100, 100-250, and above 250.

Once bus operating hours on each link of a transit route are estimated based on average operating speed and service frequency, then the bus operating cost can be computed using a proper linear regression equation for the known service condition. By summing all link operating costs, the total operating cost can be derived and then compared with the budget limit.

User Components

The transit passenger loads and their distribution in time and space are fundamental information required for evaluation and improvement of transit service. In this regard, passenger demands and their travel paths are discussed and analyzed for their incorporation into the model as major transit system variables.

Anticipated Passenger Demands. When and where people travel in the study area by bus is vital information for the meaningful analysis of transit service and determination of optimal operating policies. Passenger Origin-Destination information for the study area was collected from various sources for major bus trip generators and attractors. The information concerns average daily trips by bus, trip purposes and passenger distributions over time.

The transit system investigation requires not only existing demands but also forecasted trip requirements for the future. The future demands are usually forecasted based on such transportation planning processes as trip distribution and modal split.

In analyzing transit service as a part of the coordinated transportation system, information is usually available from two sources. One is from passenger Origin and Destination survey and the other from bus managements. Data from a bus passenger Origin-Destination survey includes bus passengers' origin and destination, bus route taken, trip purpose, boarding and alighting at bus stops, mode to and from bus stops, car ownership status and their preference of the service improvements.

Much useful information may be obtained from the bus operator which is valuable in preparing a data base for forecasting bus passenger demand. The forecast which is required for the transit model is made for each scheduled period. Information that can be collected from the bus company includes time tables, block diagrams, bus terminal operational statistics, expense sheets and fare collection statistics. A block diagram usually shows bus run number, major check points and check-in times.

A study of passenger demand profiles shows that demands are distributed over time with concentrations during morning and evening peak periods. In addition, the weekly passenger demand statistics reveals that passengers are distributed over a weekly period following a constant pattern with the highest demands on Friday and the lowest one on Sunday. This consistent pattern of demand persists within the same study area throughout the year.[8] This nature of

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[8]For passenger distributions over time, see Appendix D.

passenger distribution permits further partitioning of a transit planning cycle into smaller time intervals such as a (1) weekday morning peak period, (2) weekday evening peak period, (3) weekday off-peak period, (4) Saturday peak period, (5) Saturday off-peak period, and (6) Sunday period. The use of these schedule periods is advantageous in the sense that interrelationships of transit service among schedule periods can be analyzed in detail through the application of dynamic programming.

Generating Travel Paths. After passenger demands between major Origin-Destination pairs are known, the next task is to simulate the travel paths of transit passengers. Here, the term travel path is identical to chain as defined previously and both are used interchangeably. Since there may be more than one travel path from a given origin to a given destination, all reasonably economic paths must be considered in an actual assignment. Passengers using the same path would reevaluate their travel time and readjust their paths. As a consequence of readjustment, the transit system would inevitably come to a new equilibrium.

By simulating the travel path in mathematical equations, passenger flows are related with their actual assignment along links which have distinct properties as to operating speed,[9] travel time, service capacity and other link-related parameters.

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[9]For further information on Speed and Delay study, see (63).

In a real world problem there are a large number of passenger paths. But without losing accuracy, only those paths which might be economical to use can be easily selected by observation for the model. Furthermore, a path for a specific origin and destination needs not be completely connected by bus routes. A path can be a combination of walks, bus rides and transfers. This indicates that there may not be a direct bus route to go from one node to another. Yet, demands between nodes can be satisfied by the path which is composed of imaginary links of walks and transfers, and physical links of bus routes.

Vehicle Carrying Capacity of Streets

In previous sections, transit system components with regard to the user, the operator, and the system were discussed. In addition, the transit systems analysis is extended to a consideration of the traffic engineering aspects of the street network. Traffic engineering as it relates to bus transit operation is significant because transit movements and general auto traffic affect each other and often times bus transit has to compete with private automobiles for the use of limited roadway facilities.

In considering passenger carrying capacity of street links, the actual maximum number of buses that can pass a specific link and intersection is another important system parameter to be investigated. This is because physical traffic capacity may restrain the service frequency even though it can be provided by the available bus fleet.

The presence of a bus route on an urban street, especially the hourly volume of bus traffic during rush hours, significantly reduces the roadway traffic capacity. Therefore, it is sometimes necessary to have a traffic operational policy regarding bus volume and bus stop locations. Also, the need for exclusive bus lanes or other transit priority devices such as a bus pre-emption signal system should be evaluated. The adverse effects of bus flow on other auto traffic in congested urban areas are usually by the following reasons:

1. In and out movements from loading zones.
2. Passenger crossing at crosswalks or at midblock.
3. Passenger loading and unloading practice.
4. Blockage of turning traffic movements caused by buses standing at bus stops at a near side of an intersection.

Another important effect exerted by bus transit operation on local traffic is caused by the location and use of bus stops. Since the effect of bus stops on local traffic is quite significant, their adverse effect on traffic capacity should be considered during an initial transit planning stage. The restraining aspect of bus flows on local traffic should be incorporated in the system analysis.

Basically, bus stops affect traffic capacity at signalized intersections in the form of capacity reduction. If there is any local bus flow, the intersection capacity has to be adjusted by bus factors.

In computing bus adjusted traffic capacity of a signalized intersection, first the intersection capacity is derived as a function of approach width, percent truck, percent turning, metro area adjustment, peak hour factor and the ratio of green time to signal cycle.[10] Then the bus factor is computed using hourly bus volume, area location, approach width, parking conditions, bus stop location, i.e. nearside or farside and percent turning. This factor is to adjust the traffic capacity by multiplying the capacity derived for a specific intersection. Bus factors[11] in urban areas usually vary within a range of 0.8 to 1.3.

The above investigations of bus flow and related traffic capacity would help to determine traffic policy. Consequently, the limitations of street link capacity for adequacy of bus operation should be analyzed for the overall transit system effectiveness.

Priorities of Bus Transit Improvements

The description of major transit system components so far illuminates the complexity involved in the evaluation and improvements of bus transit service in an urban area. In connection with this complexity of the transit problem, a new concept of transit

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[10] A rational and practical method for the determination of traffic capacity has been devised in (91). Here, the capacity is defined as the maximum number of vehicles per unit of time that can be handled by a particular roadway component under the prevailing conditions.

[11] For the derivation of actual bus factors, see (5).

systems analysis is developed in this study. The new concept is to analyze and improve transit service from the systems viewpoint by evaluating concurrently the economics of the transit operator as well as the transit user.

The evaluation of the economics is based on the quantification of passenger cost, operating cost,[12] passenger revenue and vehicle ownership cost. This quantified evaluation supplemented by generally recognized priorities of bus transit improvements would assist the mass transit planner and transit industry in the formulation of an adequate transit improvement plan for an urban area.

In discussing priorities of transit improvements, the inherent problem to be noticed is the steady reduction of bus transit patronage even though bus transit is an essential means of urban transportation. Because of the reduction of patronage, the transit system in urban areas are, in general, experiencing considerable financial pressures caused by decreasing revenue and rising costs.[13]

In order to overcome these adverse financial trends, the transit industry and planners have exerted continuous efforts to eliminate operational inefficiencies on the one hand and to improve service quality to attract more people to bus transit on the other hand.

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[12]See (71).

[13]See (68).

However, present bus transit is characterized by very poor quality of service as compared with private passenger cars. Therefore, there is a definite need for improving transit service based on a logical order of priorities for improvements. For this purpose, the transit improvement priorities are discussed below.

The first priority for improvement is the reliability of bus transit service. As revealed by previous transit studies[14] and user preference surveys, one of the major disadvantages of transit service is the unreliability of service. Services should be provided on every route by running buses strictly according to schedule.

The second priority is the improvement of service quality in terms of headway. It has been observed too often that the transit service is infrequent even during peak periods or no service is provided during non-peak periods. This lack of service tends to penalize passengers causing inconvenience. Therefore, more frequent service should be provided based on the continued evaluation of the transit service requirements.

The third priority is the improvement of transit route configuration. This improvement is to ensure more convenient and quicker trips by analyzing existing routes based on the changing pattern of Origin - Destination demands and bus routing standards. The routing standards are the following:

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[14]For more information, see Nash, et.al. (35).

1. Direct with respect to geographic distribution of demands.
2. Proper connection among major activity centers.
3. Free from duplication.
4. Proper feeder service connection.

In connection with the route improvement, amenity of service and the provision of service information should be considered.

Lastly, the general improvement of service should consider the provision of clean, attractive and comfortable bus vehicles as well as bus shelters at strategic locations to protect passengers from inclement weather.

Summary

The major components of the transit model were developed in this chapter. Descriptions were made of those components related to the user, operator and the system. In addition, traffic engineering aspects of street networks and the priorities of transit system improvements were discussed.

It also provided symbolic representation of the transit network by means of links, nodes and chains. Simulation of travel paths was discussed using both physical street links for bus rides and imaginary links for passenger walking and transfering.

Other factors such as passenger demand elasticity and effects of bus transit service on traffic flow were also analyzed in conjunction with the effort to integrate transit operation with overall community transportation programs.

CHAPTER IV

DEVELOPMENT OF THE FIRST PHASE BUS TRANSIT OPERATIONS MODEL

Introduction

The main purpose of this chapter is to develop the basic structure of the first-phase transit operations model. The transit operations model is formulated into a linear programming problem. The optimal solution of the model is based on the minimization of the objective function, transit operation cost, within various constraints imposed by the passenger, the operator and the transit system. The subsequent discussion identifies each of the major elements of the model and expresses the interrelationship of system variables and parameters in precise mathematical terms.

Data Source

The mathematical development of the model first requires a sound data base. The major elements of this data base and their use in the overall study design is depicted in Figure 4. The collection of required data forms an essential part of any engineering and planning study. Bus transit studies require both time-consuming and expensive collection of data pertaining to characteristics of the bus passenger, the trip and the transit system. Bus data and related information which are essential to the application of such a planning model as proposed here, include not only the general information from conventional sources, but also comprehensive data. Some data may be difficult to obtain directly from transit surveys.

However, it may be possible to synthesize existing transit data to develop more comprehensive data for systematic analysis. For example, data concerning passenger Origin and Destination information, physical properties of street links and bus routes, and passenger fare structures are readily available from bus companies and planning agencies. In contrast, passenger load factors in terms of passengers per bus for different levels of service may not be available directly from the above sources. This is because the determination of the elasticity of demand over service requires an analysis and investigation of many related factors such as passenger preference, auto ownership, income and land use pattern.

As an extension of the theory developed in this research, the two phase transit model is applied to the practical case of a bus network in Newark, New Jersey. The corresponding data flow chart and the study design are shown in Figure 4.

In the southwest section of Newark, there have been a series of studies and data collections to improve transit services along a major route, Springfield Avenue.[1] One of the above studies pertains to an extension of the subway system currently serving downtown and northeast areas. The proposal calls for an extension of the city subway to the Irvington bus terminal which is located three miles west of the downtown area and handles the bulk of

[1] See Deutschman (78) and (104).

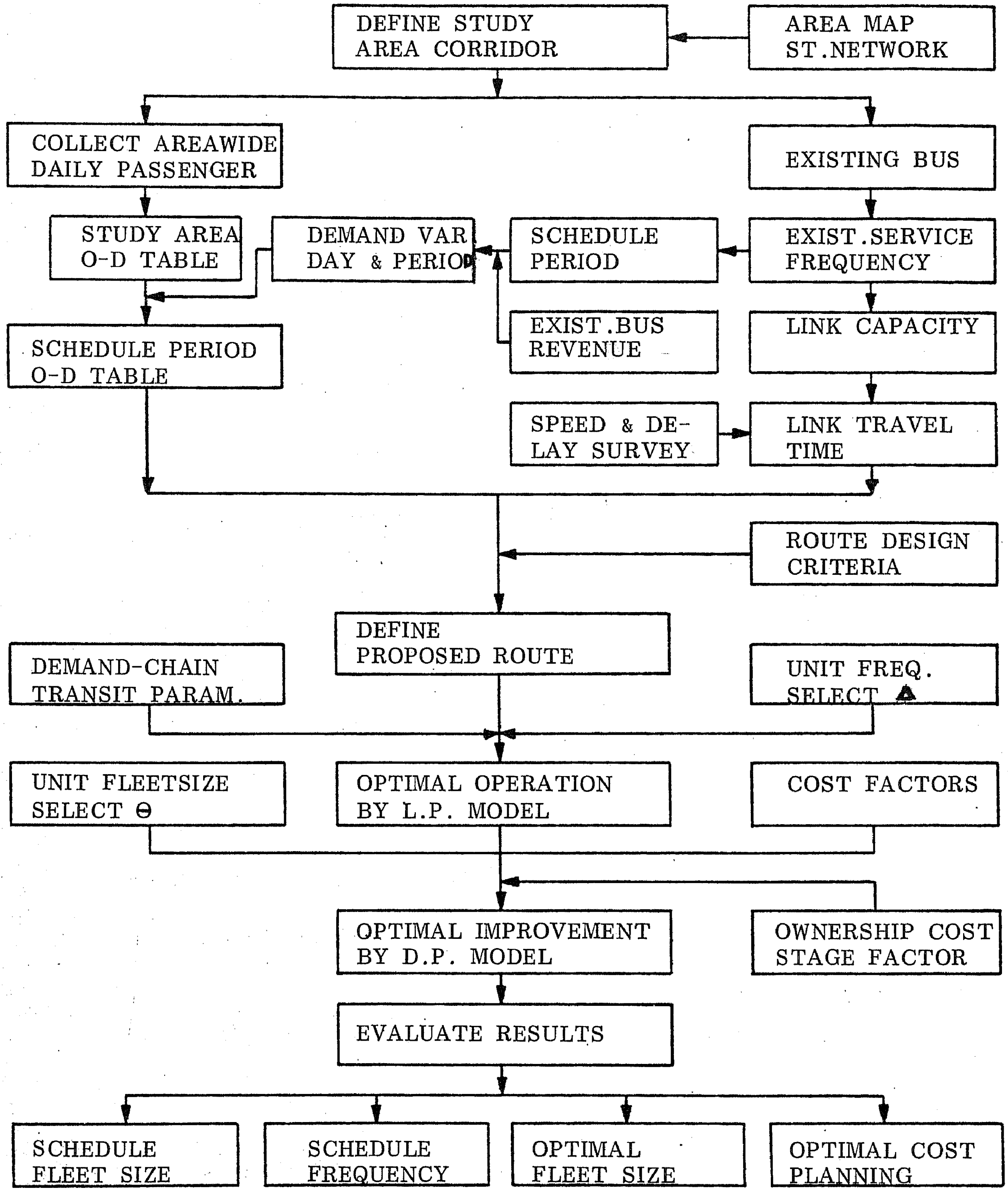

FIGURE 4. DATA FLOW AND STUDY DESIGN

downtown oriented passengers.

As a result of the transit studies, data concerning the bus system on the Springfield Avenue corridor has been collected. The collection has been made in many transit related fields, especially in areas of bus transit network configuration, bus headways for peak hours and off-peak hours, directional variations of passengers during rush hours, bus stop organization, passenger boarding and alighting information at each bus stop and most significantly bus passenger Origin and Destination information between census tracts along the corridor area.

The transit data collected for testing the transit model is first analyzed and then reduced on proper forms so that it can be directly utilized by the model. In addition to the use of existing data, some of the unavailable data, especially bus load factors for different service levels and their linear approximations have to be assumed based on past trends and engineering judgements so that the model would produce meaningful results. However, since the model has the adaptability to new data sets, the use and testing of the model should be adjusted accordingly based on new revised information whenever it becomes available.

Generation of Passenger Origin-Destination Information

The major demands selected for the application of the transit model are eleven major Origin and Destination[2] pairs between important nodes within the study network. For computational simplicity, two-way travel demands between nodes are used instead of one-way trips. However, the transit model is flexible enough to utilize one-way travel demands to represent possible directional variations in operating speed and demands.

Triangular Origin and Destination tables for each schedule period are prepared based on the passenger distributions over day, hour and week which are approximated from average 24 hour passenger information. The approximation is determined by fare collection statistics and bus schedule block diagrams which show the scheduled bus movements during each of all schedule periods within a planning cycle.

The transit trip tables are generated for six schedule periods. Table 1 shows two sample Origin and Destination tables for Sunday and Saturday off-peak periods.

Formulation of Bus Transit Operation

Bus transit operations during each schedule period of an entire planning cycle are highly dependent upon passenger needs for services, community restrictions and various constraints imposed by the transit operator. For example, the service frequency

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[2]Bus passenger Origin and Destination information is developed for census tracts along Springfield Avenue corridor in Newark, New Jersey, see (69).

(1) SUNDAY (18 hours)

O \ D	1	3	5	6	7	8	15	17	20
1		572	1472	793	1418				
3					413	674			
5					780	1650			
6						1174			
7						1561			530
8									
15									
17									
20									

(2) SATURDAY (18 hours) OFF PEAK

O \ D	1	3	5	6	7	8	15	17	20
1		432	1111	597	1080				
3					312	713			
5					588	1246			
6						887			
7						1179			400
8									
15									
17									
20									

TABLE 1 BUS PASSENGER DEMAND FORECAST DURING SCHEDULE PERIODS 1 & 2

during a particular period is governed by the socio-economic characteristics of the community which generate a certain level of transit patronage. Also, the transit operator has limited resources to allocate to transit service. Usually, the limited resources include passenger service capacity, available number of buses, restricted union contract and a limited operating budget.

Bus transit operations are usually planned within this framework of passenger demand and constraints which can be represented by an analytical relationship specified by the transit model. The model would produce the optimum system configuration after making a number of systematic comparisons of alternate transit operating policies which concern bus route, headway and fleet size when coded input enters the model.

As discussed earlier in Chapter II, the transit operations model is concerned with the optimum possible operation of bus transit during a particular schedule period which has fixed service configuration and a known average transit patronage.

The nature of the optimum transit operation is captured in the first-phase model which is formulated as a linear programming problem. The model optimizes the system performance measure, a cost-utility, which considers both the desire and interest of the transit user and the economics of the transit operator.

Structural Equations

The proposed transit operations model is essentially a large size linear programming problem. In particular, the model is similar to a network flow problem formulated in arc-chain form.[3] The size of the sample problem has sixty-six variables and thirty-eight inequalities. The basic linear programming model focuses on the transit passenger flows of the specified bus route network. Subsequently, the structure of the transit operations model is expressed by an objective function and six sets of linear equations comprising a total of thirty-eight inequalities for imposing various transit operational constraints.

Objective Function. The objective function of the first phase transit operations model is to minimize the system performance measure, the cost-utility of bus service subject to various constraints. The objective function is defined as:

Cost-utility = passenger cost + bus operating cost - passenger revenue

The decision variables specified in the model are the assigned passenger flows for each passenger demand using a particular travel path at a specified level of service for a schedule period. The passenger flow refers to the number of bus riders assigned to a chain with known costs during individual schedule periods.

Mathematically, the objective function is expressed as:

$$\text{Minimize } U^n = (W)(A)(X)^n + (L)(OC)(A)(X)^n - (F)(X)^n \qquad (1)$$

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[3] See Tomlin (44).

The notations are as defined below:

U^n = Objective function of a transit operation during nth schedule period

(W) = Row vector of passenger cost with a dimension of (1 x 1')

(A) = An incidence matrix with a dimension of (1' x dcu). The matrix has elements of 0 and 1 to define whether a chain passes a particular link or not. A more detailed definition is provided later in the section on generation of travel paths.

$(X)^n$ = Column vector of passenger flows with dimension (dcu x 1) during nth schedule period.

(L) = Inverse of load factor, scalar parameter

(OC) = Row vector of link operating cost with size of (1 x 1')

(F) = Row vector of passenger fare with size of (1 x dcu)

Here, 1', d, c and u refer link, Origin - Destination demand, chain and load factor numbers respectively. The first term, $(W)(A)(X)^n$ refers to costs incurred to passengers using the bus transit in the form of time for walking, riding and transferring. The second term, $(L)(OC)\cdot(A)(X)^n$ represents the sum of operating costs to the transit operator. This term is calculated based on the link operating cost which is correlated with transit system output such as bus-miles and bus-hours. The third term, $(F)(X)^n$ refers to the passenger revenue produced by assigning passengers over the network based on minimum total transit operation cost (U^n). When passenger demand between two nodes is satisfied, an associated fare is payed to the transit operator for the transit revenue. The structure of fare is usually based upon travel distance and zone boundaries.

Operational Restrictions. The bus transit operation during a schedule period is affected by various constraints. These constraints are usually imposed by service requirements, limited equipment and manpower, operating budget, the vehicle occupancy and other constraints to ensure path continuity.

In the following sections, each constraint is mathematically expressed to formulate a linear programming problem for the first phase transit operations model.

Passenger Demand Constraints. It is necessary to ensure that the service provided should be equal or greater than the minimum passenger demand for all origin and destination pairs within the study network. A necessary condition for these passenger demand constraints is that there exists at least one chain of links for each Origin-Destination passenger demand. The demand constraints are expressed mathematically by means of chain flows as shown in the following equation:

$$\sum_{u}\sum_{c} X_{dcu}^{n} \geqslant f \cdot r_{d}^{n} \qquad d=1, \ldots N \tag{2}$$

Where:

X_{dcu}^{n} = Number of passengers assigned on chain "c" with "u" load factor for demand "d" during schedule period "n"

f = Probability distribution factor of passenger arrivals to ensure satisfaction of demand based on a minimum confidence level.

r_{d}^{n} = The average passenger demand for demand "d" during schedule period "n"

As may be noticed, passenger demand for each Origin-Destination pair is numbered 1, 2, 3, ... N for each schedule period. Each Origin-Destination passenger demand requires one inequality and there should be as many inequality as the number of Origin-Destination demands. For the sample test case, eleven major Origin-Destination pairs are considered which requires eleven constraining inequalities.

Service Level Constraints. The constraints of service level are perhaps most difficult to understand. Their function is simply to ensure that an additional increment of service would be first provided on those bus routes which carry passengers at the higher load factor. The load factor is, as discussed earlier in Chapter II, the number of passengers per bus. The load factor is a decreasing function of bus flow since the bus transit patronage increases as the frequency of service increases but at a decreasing rate.[4] The decreasing rate is due to the limited transit market which restricts the demand elasticity over service improvements. The term, level of service is used here to refer to transit service at different load factors since the service quality can be related to load factors, especially in passenger loading and comforts.

The relationship between load factors and bus or passenger flows for known average demands can be empirically derived. How-

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[4]For more discussion and factual data, see Hartgen (89) pp. 12-25.

ever, the precise elasticity of demand due to service improvement or curtailment is not well known. For the study, two different load factors are used on the basis of passenger flows (X_{dcu}). Load factor L_1 is used for all X^n_{dc1} for $\sum_c X^n_{dc1} \leq U'^n_{d1}$ where U'^n_{d1} is the upper bound of passenger flow at the higher load factor. The constraint on bus flow due to different load factors, therefore, can be imposed as follows:

$$\sum_c X^n_{dc1} \leq U'^n_{d1} \text{ for } d = 1, 2, 3, \cdots N \tag{3}$$

where 1 represents X_{dc} with higher load factor. The total number of these constraints is equal to the number of Origin-Destination pairs. Other notations are the same as defined earlier. Passenger demands and limits of load factor 1 for each Origin-Destination pair during every individual schedule period are shown in Appendix D.

Generation of Travel Paths. The actual path of travel by a bus passenger is simulated on an individual basis rather than mass basis by connecting relevant links which represent the physical street links. Imaginary links to cover walking and transferring activities for service are also used. This path simulation is undertaken by using the concept of chain incidence on links.[5]

In order to impose other related constraints such as link passenger carrying capacity and also to formulate an objective function,

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[5]For detail, see Table II, Apendix D.

the chain flow must be related to link flow. For this purpose, incidence numbers ($E_{dc}^{l'}$) are introduced as follows:

$$E_{dc}^{l'} = \begin{matrix} 1 & \text{if chain "c" for demand "d" passes link "l'"} \\ 0 & \text{otherwise} \end{matrix} \qquad (4)$$

The notion of "incidence" of demand chains on street links is very useful to represent the actual movement of passengers in the network. Based on this notion, an incidence matrix is developed to cover all routes taken by each demand chain. Any specific chain for demand "d" can be traced on the network through the incidence matrix.

By using an incidence matrix with elements of 1 or 0, the relationship between demands and other service constraints such as service capacity, operating budget and available bus fleet can be explicitly defined.

The objective function of the transit operations model is also derived by multiplying the incidence matrix with property vectors such as passenger time, bus operating cost and passenger fares. A typical demand-chain incidence matrix is shown in Appendix D , page 187 which includes both physical and imaginary links.

Passenger Service Capacity. The basic capacity constraints concerns the limited service capacity on bus routes which are imposed by a given headway during a particular schedule period.

Here, the passenger flows of each chain are converted to link flows and then to bus flows by means of matrix multiplication. The link passenger carrying capacity is formally formulated as following:

$$L_u^{-1} \cdot (A)(X)^n \leq (C) \qquad (5)$$

The links considered here also include imaginary links. (C) is the vector notation of service capacity of each link which is represented in terms of number of buses running on each link. (C) is a column vector with as many elements as the number of links in the study network.

L_u^{-1} is the inverse of a load factor for the level of service "u". A load factor is expressed in terms of passengers per bus. If sufficient data is available to derive separate load factors for different origin and destination pairs, multiple load factors can be used for the same level of service.

Fleet Size. The operation of bus transit system is also affected by fleet size, operating budget and union contracts, etc. Fleet size is the number of buses acquired and retained to produce revenue-making system output. According to the available fleet size, the range of feasible service frequency can be determined for different schedule periods. During off-peak periods, only a portion of the total fleet is used and fleet size may not become critical. In comparison, peak periods usually have demand with high peaking characteristics and require a large fleet size. This factor can be a major constraint.

Fleet size imposes a constraint to the transit operation in the form of limited resources and can be expressed as following:

$$L_u^{-1}\ (T)(A)(X)^n \leqslant FN^n P^n \tag{6}$$

Where:

(T) = Row vector of one-way running time with size of (1 x 1')

$(X)^n$ = Column vector of passenger flows during schedule period "n"

FN^n = Fleet size during schedule period "n" (scalar)

P^n = Number of bus operating hours during schedule period "n" (scalar)

This constraint of fleet size is imposed by operating policy and has no direct relationship with the service provided during a specific period. There is one inequality of this type to ensure that total need of bus vehicles will not exceed available bus fleet and operating hours during the same period.

Operating Budget. This constraint of operating budget is also expressed in the same format as the fleet size constraint in the previous section. Here, the operating budget refers to the total direct operating cost incurred to transit operator in terms of wage, fuel, tires and other maintenance costs. These costs are well correlated with transit system output such as bus-hours and bus-miles as discussed earlier.

This constraint will have same mathematical format as (6), but with a different row vector of cost. Namely,

$$L_u^{-1} \cdot (OC)(A)(X)^n \leqslant B^n \tag{7}$$

Where:

(OC) = Row vector of operating cost with size of $(1 \times l')$

B^n = Budget limit during schedule period "n" (scalar)

One inequality of budget constraint is required so that the total operating cost during a specific schedule period be subjected to the predetermined operating budget limit.

Street Capacity Constraints. Another set of constraints may be imposed upon the transit operation by the physical traffic carrying capacity of street links or signalized intersections along the route. These constraints are redundant on most links due to the presence of link service capacity constraints. However, these constraints are pragmatic since only a limited number of buses can pass a link due to physical link capacity or traffic operational policy. Only those links that may have a capacity problem are subjected to these constraints.

These constraints have the same mathematical expression as for the link service capacity, but with different right-hand sides. The equation has the following form.

$$L_u^{-1} (A)(X) \leq (C') \tag{8}$$

Where:

(C') = Physical link capacity with size of $(1 \times l')$

There are as many inequalities of physical capacity constraints as there are links, but those links which have potential vehicle carrying capacity greater than service frequency are not affected by the constraints. If there is a link restricted by its physical capacity before the service capacity, then for computational simplicity, the right-hand side of the service capacity constraint would be replaced by the physical capacity. The replacement of the right-hand side is much simpler than having the same two linear inequalities with different values for the right-hand sides which makes one constraint redundant.

The structure of the formal linear programming problem is given in Table 2. The resulting linear programming is of the form:

$$\text{Minimize} \quad Z = CX \tag{9}$$

$$\text{Subject to} \quad E' \ X \geq D' \tag{10}$$

$$A' \ X \leq L \tag{11}$$

$$X \geq 0$$

where the vector X is the set of decision variables, C the vector of cost, D' and L the vector of right-hand side, and matrices E' and A' are coefficients of X. The structural equations are summarized in Figure 5.

Solution Method

The selection of a convenient solution method devised for the linear programming formulation depends on the type of model employed, the size of the problem and the computational facilities available to the transit planner. The transit operations problem formulated as a linear programming problem in this chapter is similar to

CONSTR. \ VAR.	X_{11}	X_{12}	X_{21}	X_{22}	X_{dc}	X'_{11}	X'_{12}	X'_{21}	X'_{22}	X'_{dc}	RHS
SERVICE ELASTICITY	1	1	0	0	. . .	0	0	0	0	. . .	$\le U_1$
	0	0	1	1	. . .	0	0	0	0	. . .	$\le U_2$ . .
LINK CAPACITY	0	0	0	0	. . .	0	0	0	0	. . .	$\le C_1$
	L_1^{-1}	0	L_1^{-1}	0	. . .	L_2^{-1}	0	L_2^{-1}	0	. . .	$\le C_2$ . .
PHYSICAL CAPACITY	0	0	0	0	. . .	0	0	0	0	. . .	$\le C'_1$
	L_1^{-1}	0	L_1^{-1}	0	. . .	L_2^{-1}	0	L_2^{-1}	0	. . .	$\le C'_2$ . .
FLEET	$L_1^{-1}T_{11}$	$L_1^{-1}T_{12}$	$L_1^{-1}T_{21}$	$L_1^{-1}T_{22}$	. . .	$L_2^{-1}T_{11}$	$L_2^{-1}T_{12}$	$L_2^{-1}T_{21}$	$L_2^{-1}T_{22}$	. . .	$\le N\ P$
BUDGET	$L_1^{-1}C_{11}$	$L_1^{-1}C_{12}$	$L_1^{-1}C_{21}$	$L_1^{-1}C_{22}$	. . .	$L_2^{-1}C_{11}$	$L_2^{-1}C_{12}$	$L_2^{-1}C_{21}$	$L_2^{-1}C_{22}$	. . .	$\le B$
PASSENGER	1	1	0	0	. . .	1	1	0	0	. . .	$\ge D_1$
	0	0	1	1	. . .	0	0	1	1	. . .	$\ge D_2$ . .
OBJECTIVE FUNCTION	c_{11}	c_{12}	c_{21}	c_{22}	. . .	c'_{11}	c'_{12}	c'_{21}	c'_{22}	. . .	

TABLE 2 LINEAR PROGRAMMING TABLEAU

Note: X_{dc} = Number of passengers for demand "d" using chain "c"
T_{dc} = The total passenger time for X_{dc}
C_{dc} = The total bus operating cost for X_{dc}

STRUCTURE OF TRANSIT OPERATIONS MODEL

Min.	$U^n = (W)(A)(X)^n + (L)(OC)(A)(X)^n - (F)(X)^n$	(1)	Objective Function
S.T.	$\sum_u \sum_c X^n_{dcu} \geq fr^n_d$	(2)	Demand
	$\sum_c X^n_{dcl} \leq U^n_{dl}$	(3)	Level of Service
	$E^1_{dc} =$ 1 if chain "c" for demand "d" pass link "1'"; 0 Otherwise	(4)	Chain Incidence
	$(L_u^{-1})\ (A)\ (X)\quad (C)$	(5)	Link Capacity
	$(L_u^{-1})\ (T)\ .\ (A)\ (X)^n \leq N^n P^n$	(6)	Fleet size
	$(L_u^{-1})\ (OC)\ (A)\ (X)^n \leq B^n$	(7)	Budget
	$(L_u^{-1})\ (A)\ (X) \leq (C')$	(8)	Physical Capacity

Min.	$Z=CX$	(9)
Subject to	$E'X \geq D'$	(10)
	$A'X \leq L$	(11)
	$X \geq O$	

FIGURE 5. LINEAR PROGRAMMING FORMULATION

that formulated by Tomlin (44) as an optimal network flow arc-chain formulation.

The linear programming formulation based on an arc-chain incidence matrix is difficult to compute because many paths between each Origin and Destination pair must be enumerated. For a practical study, a large linear programming system is recommended for the transit operations solution. Such a system is MPS/360[6] which utilizes many mathematical programming devices for efficient solution. For a large network of transit routes, the problem can be more efficiently handled by means of the Ford-Fulkerson column generating technique and the decomposition principle.[7]

Summary

The first phase, the transit operations model, was formulated as a linear programming problem in this chapter. Initially, the data source and the study design were discussed for the development of the model. Then an objective function and its elements were specified in matrix and vector form to assess the system performance of bus operations. As the result of a linear programming solution, costs incurred to the passenger and the operator were specified mathematically in the objective function.

Finally, seven sets of contraints on transit operation imposed by

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[6]For actual use of computer program, see (94).

[7]See discussion by Dantzig, et.al. (9) and Charnes, et.al. (55).

the user, the operator and the system were defined mathematically by linear inequalities. These constraints included demand, service level, chain incidence, link capacity, fleet size, budget and physical capacity of street link. Solution of the linear programming formulation was also discussed.

CHAPTER V

DEVELOPMENT OF THE SECOND PHASE BUS TRANSIT PLANNING MODEL

Introduction

The objective of this chapter is to formalize the second phase, the transit planning model. This transit planning model is structured as a dynamic programming process and it utilizes the results of the first phase transit operations model.

The transit operations model is aimed at only one service frequency state during a single schedule period. However, the transit planning cycle consists of many schedule periods with different demand and service conditions. For this reason, it is necessary to formulate a proper process to expand the transit operation from a single service frequency state of one period to the multiple frequency states during all schedule periods.

This process of expansion consists of two stages. First, the optimal service frequency state is selected from the possible range of frequency states for a given fleet size state. The chosen frequency state incurs the minimum sum of the transit operation cost and the direct route operating cost for the specific fleet size. The direct route operating cost is the cost which is not accounted for by the objective function of the first phase transit operations model and it is further discussed later in this chapter. Second, the aggregation of a single state transit operation is made through the systematic evaluation of combinations of transit operations over all schedule periods based on the dynamic programming algorithm.

Criteria for Evaluating Bus Transit Planning Alternatives

In planning a bus transit system, there are almost an infinite[1] number of transit operating alternatives. These alternatives stem from variations of transit system components such as service frequency, bus fleet size and the route locations.

Before the optimal configurations of the transit system components are sought on a proposed transit route, the feasibility of introducing the new proposed transit route should be established first. It should be based on whether the addition of the new route produces a lower total transit system planning cost or whether it does not. The analysis of the economic feasibility of the new transit route is accomplished by the use of the second phase transit planning model which is structured as a multi-stage decision process. In a multi-stage decision process, a decision at one stage affects decisions in succeeding stages. A dynamic programming model is applied to the multi-stage decision process in order to derive an optimal sequence of decisions for service frequency, fleet size and its overall transit planning cost. In order to construct a flexible and inclusive evaluation criteria of transit planning

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[1]The number of alternatives is a power function of transit variables, i.e. transit route, service frequency, fleet size and schedule periods.

such major cost components as bus operating cost, passenger cost, passenger revenue, route operating cost and finally bus ownership cost are selected as basic structural members of the criterion.

As discussed earlier in Chapter IV, the first three cost components are analyized in the first phase linear programming model. This model generates the optimum state of transit operation for a given fleet size, The information derived in conjunction with the optimum frequency state is required for the second phase transit model since it compares various transit planning costs based on all cost components.

The annual ownership cost of a bus fleet reflects the cost incurred to the transit operator. The ownership cost per vehicle has a fixed cost nature regardless of the number of times the vehicle is used. Thus, the total ownership cost increases as the required fleet size increases.

The choice of the above major transit cost components as a criterion of transit system performance represents a significant step toward the systematic assessment of transit service in urban areas. Previously, costs which are incurred by both the transit operator and the user have rarely been considered concurrently in the overall planning of a bus transit system.

Nevertheless, there is a need for discriminating one cost from another since the effects of cost components may impose different transit service conditions. For example, the monetary value[2] of passenger time may have to be weighted much lower than bus operating cost or ownership cost in places where a tight transit budget restriction is prevalent. For this purpose, the transit model incorporates cost weighting factors as discussed in a later section of this chapter.

In summary, the criterion of the transit planning model is the transit planning cost which covers all salient cost elements of a transit service for both the individual schedule period and the overall weekly planning cycle.

Dynamic Programming Process

The procedure of dynamic programming is briefly described here to relate its application to the bus transit planning model presented in this chapter. In the discussion of a dynamic programming problem, a stage refers to one of the decision points which, in sequence, comprise the multi-stage decision problem. Meanwhile, states are the various possible conditions in which the system may find itself at a particular stage of the problem. In the transit planning model, stages are transit schedule periods while states associated with each stage are fleet sizes which the transit system may have at that schedule period. The dynamic

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[2]For the case study, a bus passenger time value of $2.40 per hour was used. For further discussion see Lisco (98).

programming approach shows that one can compare the transit planning cost and benefit of moving to another state, given that the system is in a particular state at a particular decision stage. Using this approach, the user can make the optimal decision at each decision stage to yield maximum transit system benefit. The course of optimal decision at each decision stage can be traced when the decision process is completed for all decision stages.

The bus transit planning problem is characterized by basic features of a simplified dynamic programming problem in the sense that:

1. The problem can be divided up into a sequence of stages with a policy decision required at each stage.

2. Each stage has a set of states which are transformed to other states in the next stage by the decision made in the present stage.

3. The optimal policy for the remaining stages is not affected by decisions made in previous stages.

4. A recursive relationship exists between any two succeeding stages that identifies the optimal policy at the present stage given that the optimal policy for each state for all previous stages are known.

Based on these basic features of typical dynamic programming problems identified, the formulation of the transit model is further discussed here. In this model, the transit planning cycle of a week is divided up into six stages of schedule periods. The transit policy decision at each schedule period is the determination of whether additional bus vehicles are to be introduced into the system. By the addition of these vehicles, the service frequency of the transit system may be increased.

Another feature of the model concerns various fleet sizes associated with each decision stage, a schedule period. Once a specific decision is made during a schedule period, then the existing fleet size is transformed into another fleet size for the next schedule period according to the decision on additions to the bus fleet. Based on two computational properties, that is, the independence of previous decisions on the overall optimal decision path, and the recursive relationships of decisions between two succeeding stages, the transit model computes the optimal solution proceeding backward starting from the last stage. The model proceeds with the derivation of the optimal decision (additional fleet) stage by stage, each time finding the optimal policy for each state of fleet size of a specific schedule period until it completes the whole planning cycle.

Design of Stages

As a first step toward formulation of the second phase transit planning model, the nature of the multi-stage decision process of a bus transit planning is to be recognized.

An observation and analysis of the existing bus transit schedule reveals that the service can be homogenously specified for different times of day, i.e. morning peak period, evening peak period and off-peak periods as well as for different days of the week, i.e. average weekday, Saturday and Sunday with distinct weekly cyclical pattern. Consequently, the schedule periods are designed as decision stages in the dynamic programming model with a total of six schedule periods.

The fundamental assumption underlying the schedule period is that each schedule period has homogeneity in passenger travel demands, trip purpose and trip makers' characteristics during the same period. This concept of the schedule period is analogous to the design hour volume or peak hour volume for highway or intersection design. Design hour volume or peak hour volume[3] is traffic volume measured in the number of cars during a unit time period. These volumes are used in designing a highway or an intersection to satisfy traffic demand at a certain confidence level, for example 95 percent of demand times, even though the traffic volume is distributed widely over time and area. In designing an efficient highway facility more than one volume may be used to take traffic variations into account. For example, three different traffic volumes can be efficiently used for the economic design for an intersection. They are morning peak hour volume,

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[3]For more information see (86) and (63).

evening peak hour volume and mid-day off-peak volume to represent variations of traffic volumes for all periods.

Likewise, the bus transit demand and service variations are represented by six schedule periods. For better accuracy, these schedule periods can be further refined in sufficient detail by increasing the number of schedule periods. The use of the schedule period enables the determination of the sequence of the optimal decision at each decision stage. The sequence of the optimal decisions concerns itself with the feasibility of a new route, the optimal service frequency and the optimal fleet size which together determine the minimum annual total transit planning cost for the study area.

Once the schedule cycle of a week is further partitioned into individual schedule periods, each schedule period is ordered according to the passenger demand density. The density is expressed as a passenger concentration during a unit time, namely as passengers per hour. This rank ordering of schedule periods according to their density[4] is to represent the difficulty of reducing bus fleet size without financial losses once increases are introduced to the existing system. In other words, the rank ordering of schedule periods ensures that if a bus is introduced to the existing system,

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[4]For bus passenger density of selected bus lines in Newark, New Jersey during peak and non-peak weekday periods, see Deutschman (78).

it should be in the system, thereby restricting the freedom of bus fleet as a system variable. The ordering is just a practical consideration to ensure an extensive investigation of all system variables that have more freedom than the number of new buses before any attempt is made to increase the fleet size. One example of the transit system variable which has more freedom of adjustment than fleet size is the service frequency that can be provided by the existing bus fleet and manpower. In reality, it is more economical and flexible to adjust service frequency than fleet size if the service frequency can be provided by the unused existing fleet.

The actual arrangement of schedule periods are in the order of Sunday, Saturday off-peak period, weekday off-peak period, Saturday peak period, weekday A.M. peak period followed by weekday P.M. peak period which has the highest demand density. The beginning and ending of each schedule period and its duration are shown in Appendix D.

Design of States

The concept of state of a schedule period refers to conditions of two major system variables that can be modified during the same schedule period. They are states of service frequency and states of the retained fleet sizes. The first is the condition of service frequency provided on the proposed new route while the second refers to that of fleet size operating on the same route during the same schedule periods. In the dynamic programming formulation, fleet sizes are states associated with each schedule period. The fleet size imposes a boundary condition on service frequency in terms of maximum service frequency.

In choosing a service frequency for a specific demand during a particular schedule period, a uniform increment of frequency is selected to define different states of service frequency. In this study, for example, an increment of fifty bus runs per period is used. Now let delta (Δ) denote one increment, that is, fifty additional bus runs, then service frequency states with O, Δ, 2Δ, 3Δ, . . . $M\Delta$ would have 0, 50, 100, 150, ... 50M service frequencies during the same schedule period. The numerical value of Δ can be chosen at will, thereby, the accuracy of this state of service frequency can be further refined if it is necessary.

The second state of fleet size can be similarly represented to define the fleet sizes retained for the specific service frequency. To depict the state of fleet size for each different period, a uniform increment is again used which can vary as follows:

$$FN = 0,\ \theta,\ 2\theta,\ 3\theta,\ .\ .\ .\ k\theta$$

where FN is fleet size and θ denotes the uniform increment of fleet size. The increment can be selected arbitrarily. $\theta = 5$ is used for the sample computer run.

The number of buses available has no direct relationship with service frequency except that the fleet size provides the boundary condition of the potential service amount. Each state of fleet size can provide a different range of frequencies as long as the service does not exceed the limited equipment expressed as available bus-hours.

For example, if a bus can provide ten dispatches during a schedule period and the service frequency increment of ten is used, then a fleet of five buses can provide service frequency of 0, 10, 20, 30, . . . 50 bus runs during the same period.

Once additional buses are added to the transit system during any schedule period, they remain in the system. Therefore, a retained fleet size is either constant or increasing starting from the lowest passenger density to the highest one. However, service frequency can vary and it can be even reduced if necessary regardless of the ordering of schedule periods.

There is a trade-off in adding more buses on a route. If more buses are added, then more bus service can be provided. However, the returns from having bigger fleet size are not always paid off since the ownership cost and the operating costs may increase more rapidly than the benefits derived from the higher service.

The fleet size that has to be retained to provide a specific service of frequency is derived based on the following formula:

$$\text{Fleet size} = \frac{\text{Round Trip Time (Min.)}}{\text{Headway (Min.)}}$$

$$\text{Round Trip Time} = \frac{\text{Route Length}}{\text{Schedule Speed}} + \text{layover time}$$

Once a round-trip is computed based on schedule-speed, route length and layover time, the number of vehicles required to operate a given headway is estimated by dividing the round-trip time by the headway. The bus headway is the time spacing between two successive buses and is calculated by dividing the time duration by the number of bus runs.

Let FN = Fleet size

T_r = Round trip time

H = Headway in minute

L_r = Route length in miles

V = Schedule speed in miles/hour

L_t = Layover time

Δ = Service frequency increment (bus runs)

K = Number of service frequency states

Pn = Number of hours in schedule period

Then,

$$FN = \frac{L_t + \dfrac{60 \times 2 \times L_r}{v}}{H}$$

$$= \left(L_t + \frac{120\ L_r}{v}\right) \Big/ \left(\frac{60 \times Pn}{k\Delta}\right)$$

For practical purposes, the value of fleet size is rounded to the next higher integer. The selection of fleet size for a given service frequency or the choice of service frequency state for a given fleet size is determined using the above formula.

Direct Route Operating Cost

The direct route operating cost is defined as cost which is not included either in the transit operation cost or in the vehicle ownership cost. This cost has to be considered separately because it may not be accounted for by the objective function of the first phase linear programming model. The new transit service may be used only partially in the optimal transit operation even though the new service incurs a fixed amount of operating cost measured as a function of the service frequency provided.

The direct route operating cost can be readily computed after the linear programming model yields the optimal transit operational configuration and associated passenger flows. The operating cost which is not included in the objective function is derived by multiplying the unused part of the new service frequency with a unit bus operating cost.

The direct route operating cost implies two important bus transit planning considerations. One consideration is that, for existing transit service, only operating cost for the used service should be considered. In other words, there can be unused service whose cost is not directly included in the total transit operation cost. The second consideration is that, for the new transit service, the total bus operating cost should be considered in full measure even though there may be unused service. The differentiation in operating cost betweeen the existing service and the new service is to reflect that the existing service should be modified according to the optimal solution while the new service should be added only in a necessary amount. The direct route operating cost also indicates that by the introduction of new service into the existing system, the existing service should be accordingly modified.

Let ROC denote the direct route operating cost, then the following expression can be made:

$$ROC = (OC) \times (C + M\Delta - L^{-1}X) \quad \text{for all links covered by the new route and } ROC \lesssim OC \times M\Delta \tag{12}$$

Where:

(OC) = Link operating cost vector with dimension of (1 x 1')

(C) = Existing link service frequency with dimension of (1' x 1)

$M\Delta$ = Mth service frequency state

$(L^{-1}X)$= Assigned optimum bus link flow with dimension of (1' x 1)

$L^{-1}X$ can be derived from optimal passenger flows as follows:

$$L^{-1}X = L_u^{-1} \ (A) \ (X) \tag{13}$$

where notations are same as discussed in Chapter IV.

Bus Ownership Cost

The annual bus ownership cost of acquiring and retaining bus vehicles must be known to determine the transit operational and planning policies using the two phase joint linear and dynamic programming model.

The ownership cost covers all costs incurred by the fleet of buses including the expenses for principal payment, interest, taxes, insurance, vehicle registration and depreciation. This ownership cost is analogous to the fixed cost of the inventory cost model which is often referred to as a setup cost while the operating cost is variable cost directly proportional to the amount of transit service output.[5] Consequently, an ownership cost is affected only by the number of vehicles, the purchase price, the vehicle operating life-span and salvage value and not by the amount of operation.

The annual ownership cost of a specified fleet size can be easily determined by an analytical approach. This approach computes the constant annual cost flow of an investment on the bus fleet for its life-span by a long accepted formula of engineering economics. This annual cost analysis usually includes three major items such as depreciation, interest and other expenditures. Mathematically the annual ownership cost can be expressed as follows:

$$AOWC = \left((PR - S) \times \frac{I}{(1 + I)^t - 1} + PR \cdot I\right) \cdot VN \qquad (14)$$

Where:

AOWC = Annual ownership cost

PR = Purchase price of bus vehicle

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[5] In an inventory model, the cost of ordering or manufacturing is usually composed of two parts, one which is proportional to the amount ordered and another which is constant.

I = Rate of return demanded on investment

S = Net salvage value of the equipment at the end of its estimated life

t = Estimated service life of vehicle

VN = Number of vehicles.

This cost reflects the constant cash flow by taking into account purchase price, rate of return, the net salvage value at the time of replacement and other financing service charges.

Weighting Factor

In determining the optimal transit operation for a specific service condition during a particular stage of schedule period or for planning overall optimum transit system, many variables have to be introduced to the transit model. Accordingly, during the planning phase there is a need to consider how different policies will be affected by the variables in the system. This need requires flexibility for the transit planner to weight variables differently. For example, in some cases, bus operating cost has to be weighted higher than the passenger time value since the former is far more restrictive for the service improvement than the latter in a short planning period.

For this reason, two sets of weighting factors are introduced to be incorporated into the transit model. The first set of weighting factors are for basic cost variables of the transit system such as bus operating cost, passenger cost and passenger revenue while the second set pertains to the annual bus ownership cost. Different degrees of bus usages may require different weighting factors so that ownership cost can be charged according to its usage. In order to weight the ownership cost, both retained fleet size in the system and the time when the additional buses are introduced into the system should be known. Once the overall fleet size and the number of buses introduced to the system during each schedule period are known, the numerical values of weighting factors for ownership cost are computed according to the number of hours of bus use during a year. The actual values used for the application of the model are shown in both Appendices D and E.

Transit Planning as a Dynamic Programming Problem

The mathematical expression of the second phase transit planning model is based on the concept of state and stages as well as the recursive relationahip between succeeding stages. As a basic input to the dynamic programming model, the results of the first phase, linear programming model are utilized for the derivation of the optimal transit planning configuration.

If f^n denotes a state of fleet during the schedule period "n", then the transit planning process can be expressed as interrelated relationship among multi-stage transit decision process. The overall optimum transit planning cost and its associated transit planning configurations are derived through the following recursive relations.

$$V^{n-1}(f^{n-1}) = \underset{f^{n-1}}{\mathrm{Min}} \left[(\overset{*}{U}{}^{n-1} + ROC^{n-1}) + D^n(f^{n-1}, f^n) + \overset{*}{V}{}^n(f^n \quad) \right] \qquad (15)$$

Where:

$V^{n-1}(f^{n-1})$	=	Minimum achievable transit planning cost at schedule period n-1 for all schedule periods > n-1, given that the fleet size is in a state of "f^{n-1}" at schedule period n-1
f^{n-1}	=	Fleet size states at schedule period n-1
$\overset{*}{U}{}^{n-1}$	=	Minimum transit operation cost at schedule period n-1 given that the fleet size is in a state of f^{n-1}
ROC^{n-1}	=	Direct route operating cost at schedule period n-1
$D^n(f^{n-1}, f^n)$	=	Cost of introducing a bus fleet at the end of the schedule period n-1 for permissible transition
$\overset{*}{V}{}^n(f^n)$	=	The minimum transit planning cost of all schedule periods $\geq$ n.

This functional relationship applies to all schedule periods and to all permissible transition of fleet size.

In considering the boundary condition of a transit planning, let N_s denote the last schedule period, then the schedule period for $n \geq N_s+1$, the optimum transit planning cost is defined to be

$$\overset{*}{V}{}^n(f^n) = 0 \text{ for } n \geq N_s+1 \tag{16}$$

As discussed in the previous section, there may be a situation where cost variables must be weighted differently according to bus transit policy which represents a prevalent transit budget or other characteristics of the community. This discriminating treatment of cost factors can be accomplished in the transit planning model by introducing the associated weighting factors as shown below.

$$V^{n-1}(f^{n-1}) = \underset{f^{n-1}}{\text{Min}} \quad (a\overset{*}{U}^{n-1} + bROC^{n-1}) + e^{n}D^{n}(f^{n-1}, f^{n}) + \overset{*}{V}{}^{n}(f^{n}) \tag{17}$$

where a and b are weighting factors for the transit operation cost and route operating cost respectively while e refers to the weighting factor for bus ownership cost for the schedule period "n".

The transit planner may have a further reason to restrict transit operation between any two schedule periods because the revenue equipment is unavailable due to repairing or service required for peak hour operation. The transit operator may even have a policy to smooth out frequencies during schedule periods. This constraint can be easily imposed on the objective function by specifying that $D(f^{n-1}, f^{n})$ must be less than a certain predetermined amount. This smoothing out of the budget can be formally expressed as follows:

$$V^{n-1}(f^{n-1}) = \underset{f^{n-1}}{\text{Min}} \quad (a\overset{*}{U}^{n-1} + bROC^{n-1}) + e^{n}D^{n}(f^{n-1}, f^{n}) + \overset{*}{V}{}^{n}(f^{n}) \tag{18}$$

for only $D^{n}(f^{n-1}, f^{n}) \lesssim SB^{n}$

where SB^{n} referes to a specific schedule budget limit during schedule period n.

The computational procedures for the second phase transit planning model can be outlined in two steps as shown in Figure 6. First step is to vary the service frequency during each schedule period using the uniform frequency increment, Δ and obtain the optimum transit operation cost for each service frequency. The optimum transit operation cost is derived by the linear programming model for every frequency state of each schedule period.

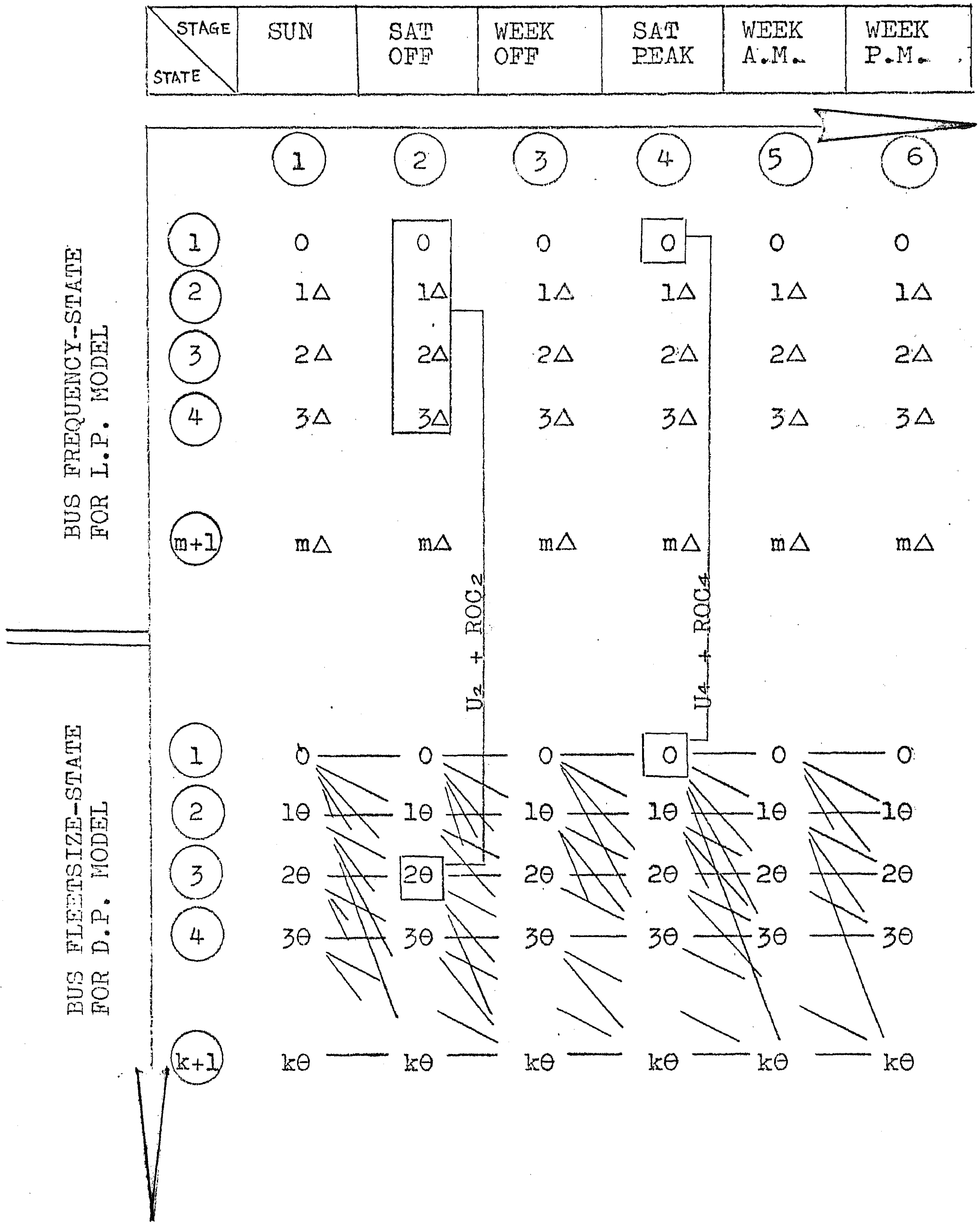

FIGURE 6 A JOINT LINEAR AND DYNAMIC PROGRAMMING MODEL

As a second step, a state of retained fleet size is chosen starting from the last stage. Then, based on the same fleet size, duration of schedule period, bus runs per period and the optimum bus flow, the range of feasible service frequency is set. From this range, the frequency yielding the best transit operation cost is found. Then, starting from the last schedule period, the transit operation cost, route operating cost and the decision cost of adding more buses are summed up and stored in the proper stage to go to a state in the next stage. The transformation of states are made by the decision of adding more buses at each stage of schedule period. The feasible transformation of fleet size and the feasible paths of decisions for any successive periods can be best illustrated by the dynamic programming structure shown in Figure 7.

The number of fleet states for a particular stage can be adjusted as necessary using the fleet size formula in the previous section.

One of the characteristics of dynamic programming is that the solution procedure usually begins by finding the optimal policy for each state of the last stage of the schedule period. A computer program is developed to compute the transit planning cost by approaching the optimum solution backward starting from the last stage.[6] The listings of computer programs are shown in Appendix C.

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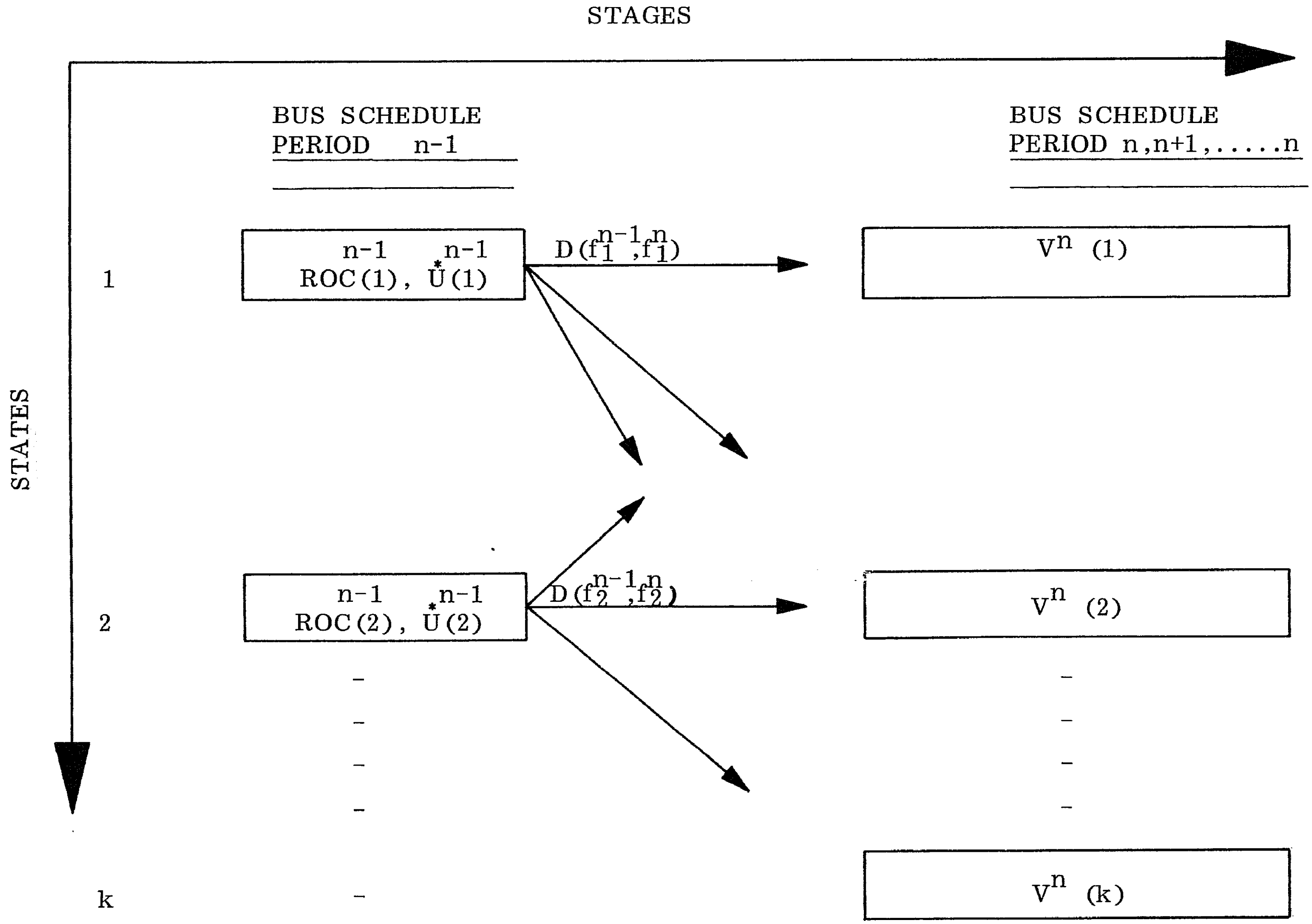
[6]The author would like to thank Mr. Donald Cohen of Newark College of Engineering for his aid in the programming stage of the research.

DYNAMIC PROGRAMMING STRUCTURE



$$V^{n-1}(f^{-1}) = \underset{f^{n-1}}{\text{Min}} \quad (a\overset{*}{U}^{n-1} + bROC^{n-1}) + e^n D(f^{n-1}, f^n) + V^{*n}(f^n)$$

FIGURE 7 BUS TRANSIT PLANNING MODEL

Once all optimum paths associated with every state of the initial stage are found, then the overall optimum path of decision for the entire planning cycle can be derived. The overall optimum solution will give the optimum total transit planning cost with all necessary information on the optimum transit system configuration. The optimum solution derived by the two phase transit model provides useful information for the transit planner to evaluate the affects of the bus transit system modification on costs as well as benefits to the community. A summarized dynamic programming formulation is shown in Figure 8.

Summary

In this chapter, the second phase transit planning model was formulated as a dynamic programming problem to extend single period transit operations to multiple schedule period transit planning.

In the first section, the criterion for the evaluation of a transit system during multiple periods was discussed in conjunction with the single period transit operation. The following sections offered the basic elements of the second phase transit planning model as it was structured in a dynamic programming algorithm. As basic elements, decision stages of the model and the associated states were designed to illustrate the structure and the operation of the model. Then, more discussions were provided for the additional cost consideration in the second phase model which incorporated the route operating cost and the bus ownership cost with the re-

STRUCTURE OF BUS TRANSIT PLANNING MODEL

ROC =	$(OC) \times (C+MW - L^{-1}X)$ for all links cover by the new route and $ROC \leq OC \times MW$	(12)	Route Oper. Cost
$L^{-1}X = L_u^{-1}(A)(X)$		(13)	Assigned Bus Flow
$AOWC = ((PR-S) \times \frac{I}{(1+I)^{t-1}} + PR \cdot I) \cdot VN$		(14)	Ownership
$V^{n-1}(f^{n-1}) = \underset{f^{n-1}}{\mathrm{Min}}(\overset{*}{U}^{n-1} + ROC^{n-1}) + D^n(f^{n-1}, f^n) + \overset{*}{V}^n(f^n)$		(15)	Recursive Relationship
$\overset{*}{V}^n(f^n) = O$ for $N \geq N_s+1$		(16)	Boundary Condition
$V^{n-1}(f^{n-1}) = \underset{f^{n-1}}{\mathrm{Min}}(a\overset{*}{U}^{n-1} + bROC^{n-1}) + e^nD^n(f^{n-1}, f^n) + \overset{*}{V}^n(f^n)$		(17)	Weighted D.P.
$V^{n-1}(f^{n-1}) = \underset{f^{n-1}}{\mathrm{Min}}(a\overset{*}{U}^{n-1} + bROC^{n-1}) + e^nD^n(f^{n-1}, f^n) + \overset{*}{V}^n(f^n)$ for only $D^n(f^{n-1}, f^n) \leq SB^n$		(18)	Budget Constraint

FIGURE 8 DYNAMIC PROGRAMMING FORMULATION

lated weighting factors for a flexible transit policy decision. Finally, in the last section, the mathematical equations were developed to formalize the transit planning concept in precise terms. A set of equations specified a recursive relation of the model, boundary conditions, cost weighting and budget leveling.

CHAPTER VI

APPLICATION OF THE BUS TRANSIT MODEL

Introduction

This chapter presents some of the potential applications of the methodology developed in this research. It illustrates the capabilities of the transit model through its application to a real world problem and an evaluation of results.

The new transit planning technique developed allows the study of numerous transit operational questions and transit planning problems which would help decide optimal transit operating policies and improvement alternatives. The case study presented here investigates major transit system elements which are critical to both the transit user and the operator. From the evaluation of results, types of decisions that the transit planner can make and the kinds of transit planning problems that the model can address are discussed.

Application of the Model

The context in which this case study is conducted, is the Springfield Avenue corridor in Newark, New Jersey as discussed earlier in Chapter IV. The input data to the model is traffic and transit data for the study corridor for a cycle of one week period.

Eleven passenger origin and destination pairs among major transit nodes in the area are considered for the application of the model. Meanwhile, fourteen street links are used to specify the existing and proposed bus route structure. In addition, altogether, thirty three demand-chains and two load factors are taken into consideration to realize passenger trip desires. Passenger demands used in the model are two-way demands for each Origin and Destination pair for six schedule periods within the weekly transit planning cycle.

The input data is coded as shown in Appendix E for computer programming routines. These routines are specifically designed to operate the transit planning model for an actual application.[1] The computer programming logic, flow charts and computer programs are attached in Appendix C.

The first part of the data presents the required input to the first phase transit operations model formulated as a linear programming problem. The data consists of five major blocks as discussed in Chapter IV. The second part of the data specifies the required input to the second phase transit planning model structured as a dynamic programming process. The number of stages and states involved in conjunction with service frequency and fleet size are identified here. In addition, results of the first phase model are coded as basic input for the second phase dy-

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[1]The actual application of the model was performed by the TSOS system of Newark College of Engineering and the MPS/360 Linear program of Princeton University.

namic programming model. This input identifies the optimal transit operation during one time period. For example, each input represents the optimal transit operation cost for the corresponding individual state of service frequency.

Evaluation of Cost Impacts of the Optimum Transit System

The optimum transit system configuration is the final result of the model application. The optimum system is intended to provide a transit service which is optimal in terms of transit routing, headway and fleet size for all service periods.

As indicated in the case study which is tested on a computer system[2] in the context of a real world problem, the two phase joint transit model is capable of analyzing the cost impact of numerous transit system alternatives. The transit alternatives arise when the transit planner varies transit system configurations. In fact, there are almost an infinite number of variations of transit routes, headways and fleet sizes for each individual schedule period. Among these variations, a certain alternative is found to provide better service than others in terms of the annual total transit planning cost, a criterion developed for a transit system evaluation.

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[2]For flexibility in the use of package linear programs, computer input coding is also provided for other package programs such as SSLP of RCA and LINPRO of Amos Tuck School of Hanover, New Hampshire.

Using this criterion and the results from the model, the transit system alternatives can be compared with one another. This comparison is very useful when planning a new bus route or re-evaluating an existing route since the transit planner needs an estimate of the potential cost savings that any of the new system variations would produce. For this reason, the cost savings of the optimal transit system is estimated within the accuracy of increments[3] selected for the analysis of service frequency and fleet size. The incremental costs resulting from the variations of a route, headway and fleet size are derived based on the computer output of the transit model and tabulated in Tables from 3 through 7, inclusive.

The principal comparison made between the optimum transit system and another system configuration is the total transit planning cost for one whole year of transit service within the study area. As already discussed, the total planning cost includes passenger cost, bus operating cost, passenger revenue and annual bus ownership cost.

The optimal system is proved superior to another sub-optimal transit system with regard to the annual total transit planning cost. These tables showing cost comparisons among various alternatives can be readily understood. However, in order to gain a full

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[3]This refers to both service frequency and fleet size increments.

understanding of the impacts of the optimal system on transit cost, a further detail review is made here.

The cost comparison illustrated in Table 3 examines the cost implications of abandoning the new proposed route. As indicated in Table 3, a significant cost difference is recognized. For example, the installation of the new transit route with the optimal headway and fleet size yields a significant reduction of cost over the existing system. The percentage reduction of the total transit planning cost for the entire period is 26.6 percent of the existing cost. This comparison indicates that by adding the new route to the existing transit routes and by operating the optimal service frequency and fleet size on the route recommended by the transit model, a total 26.6 percent of the annual total cost, that is, the sum of passenger cost, bus operating cost, passenger revenue and the annual bus ownership cost, can be saved. This cost reduction clearly recommends the installation of the new route at the specific location with the recommended service capacity as to service frequency and fleet size as derived from the optimal transit system analysis.

Of the six schedule periods, schedule periods 4 (weekday off-peak period) and 5 (weekday A.M. peak period) have most significant cost savings of 77.5 and 57.9 percent cost savings respectively over the same existing schedule periods. The optimal fleet size along the new route is 10 buses from Sunday period to Saturday off-peak period, and 15 buses during A.M.

TABLE 3

COST COMPARISON 1

OPTIMAL FLEET SIZE AND OPTIMAL SERVICE FREQUENCY
VERSUS
EXISTING SYSTEM

No.	Schedule Period	Optimal Fleet Size	Optimal Transit Planning Cost	Transit Planning Cost for Existing System	Cost Ratio	Percent Difference
1	Sun.	10	1,930	2,287	1.185	18.5%
2	Sat. Off.	10	974	1,192	1.224	22.4%
3	Week Off	10	4,974	6,190	1.245	24.5%
4	Sat. Peak	10	129	229	1.775	77.5%
5	A.M. Peak	15	922	1,456	1.579	57.9%
6	P.M. Peak	15	1,495	1,847	1.236	23.6%
	Annual Cost		10,424	13,201	1.266	26.6%

Note: All costs are in Thousand Dollars.

peak and P.M. peak periods.

Table 4 shows the interrelationship between optimal service frequency and base fleet size during every individual schedule period. As defined earlier, the base fleet refers to the number of buses introduced to the existing transit system to operate on the new route at the start of schedule period 1, Sunday period. The base fleet is intended to be utilized during all planning cycles and can be used as a good basis for determining a fleet size. As seen in the table, the service frequency is represented by the frequency state number which is a multiple of a unit service frequency. The total transit planning cost for different base fleet is based on the optimal transit operation costs and the annual bus ownership costs. As observed from the row of operation cost, the annual total transit operation cost decreases as the base fleet increases. This is because as base fleet increases, the service frequency that can be provided by the base fleet during any schedule period can be increased, thereby increasing the transit capacity of a route which may be more direct and economical to use for both the transit user and the operator.

However, the total transit operation cost decreased continuously up to a certain limit and then stays constant. This is because service provisions beyond the demand requirements tends to waste available bus fleet capacity even though only adequate headway is provided during schedule periods.

TABLE 4

OPTIMAL FREQUENCY FOR DIFFERENT BASE FLEET

Scd. Prd. \ Base Fl't.	0	5	10	15	20	25	30	35	40
P.M. Peak	0	0	0	50	50	50	50	50	50
A.M. Peak	0	0	0	50	50	50	50	50	50
Sat. Peak	0	0	50	50	50	50	50	50	50
Week Off	0	50	100	100	100	100	100	100	100
Sat. Off	0	100	150	150	150	150	150	150	150
Sun	0	100	150	150	150	150	150	150	150
Ann. Tran. Oper. Cost	13,201	11,868	11,259	10,357	10,357	10,357	10,357	10,357	10,357
Ann. Bus Own. Cost	0	25	50	75	100	125	150	175	200
Annual Tot. Cost	13,201	11,893	11,309	10,432	10,457	10,482	10,507	10,532	10,557

Note: All costs are in thousand dollars
Frequency is bus runs per period

In contrast, the annual ownership cost of bus vehicles increases monotonically as the base fleet increases in its number. This is because a certain amount of ownership cost is required to retain a specific fleet size regardless of their actual use on bus routes.

Another cost comparison between the optimal transit planning cost and the sub-optimal transit planning cost which is based on variable fleet size and the optimal service frequency provided by the corresponding fleet size is tabulated in Table 5. Here, the optimal transit planning cost, of course, refers to that which is based on the optimal service frequency and fleet size derived from the model. As realized from the column of the differential cost of the table, the total transit planning cost of the sub-optimal system is varying depending upon the fleet size being retained for the entire schedule period. The sub-optimal transit planning cost for a specific fleet size includes the annual total transit operation cost and the bus ownership cost for the given fleet size. The transit operation cost considered here is for the optimal service frequency that can be provided by the corresponding fleet size.

The highest difference of the total transit planning cost is between the optimum transit planning cost and that for fleet size of zero. The difference is 26.6 percent and it is the same with the cost difference already discussed in the previous Table 3.

One interesting implication of the incremental cost in conjunction with fleet size is observed for a fleet size of 15. The

TABLE 5

COST COMPARISON 2

INCREMENTAL FLEET SIZE AND OPTIMAL SERVICE FREQUENCY
VERSUS
OPTIMAL FLEET SIZE AND OPTIMAL SERVICE FREQUENCY

Fleet Size	Transit Planning Cost	Optimal Transit Planning Cost	Cost Ratio	Percent Difference
0	13,201	10,424	1.266	26.6%
5	11,893	10,424	1.141	14.1%
10	11,309	10,424	1.085	8.5%
15	10,432	10,424	1.001	0.1%
20	10,457	10,424	1.003	0.3%
25	10,482	10,424	1.006	0.6%
30	10,507	10,424	1.008	0.8%
35	10,532	10,424	1.010	1.0%
40	10,557	10,424	1.013	1.3%

Note: All costs are in Thousand Dollars

difference between the optimal transit planning cost and that for a fleet size of 15, even though it is very small, indicates that the optimal fleet size for the overall period may not necessarily be the optimal fleet size during different periods. This is especially true when a bus fleet can be introduced into the existing system at the middle of the transit planning cycle at a reduced ownership cost. The fixed optimal fleet size of 15 for the entire planning cycle will cost 0.1 percent more than the optimal fleet size which is flexible to vary with 10 buses for schedule periods from Sunday through Saturday peak periods and 15 buses for A.M. and P.M. peak periods. The fleet size that produces the second highest differential transit planning cost is fleet size during A.M. peak periods which cost 14.1 percent more of the total transit planning cost.

In conjunction with service frequency provided by the optimal fleet size during each schedule period, another interesting cost implication of service frequency is presented in Table 6. Table 6 illustrates the relationship between the optimal transit planning cost and that for maximum service frequency provided by the optimal fleet size during individual schedule periods. As shown in the row of schedule period 3, this schedule period has the optimal fleet size of 10 and the optimal service frequency of 100 which jointly incur the optimal transit planning cost for the same schedule period. The optimal cost is 1.8 percent less than that for the maximum service frequency that can be provided by the

TABLE 6

COST COMPARISON 3

OPTIMAL FLEET SIZE AND OPTIMAL SERVICE FREQUENCY
VERSUS
OPTIMAL FLEET SIZE AND MAXIMUM SERVICE FREQUENCY

No.	Schedule Period	Optimal Fleet Size	Optimal Frequency	Optimal Transit Planning Cost	Maximum Service Frequency	Transit Planning Cost For Max. Frequency	Cost Ratio	Percent Difference
1	Sun.	10	150	1,930	150	1,930	1.000	0.0%
2	Sat. Off	10	150	974	150	974	1.000	0.0%
3	Week Off	10	100	4,974	150	5,061	1.018	1.8%
4	Sat. Peak	10	50	129	50	129	1.000	0.0%
5	A.M. Peak	15	50	922	50	922	1.000	0.0%
6	P.M. Peak	15	50	1,495	50	1,495	1.000	0.0%
	Annual Cost			10,424		10,511	1.008	0.8%

Note: Frequency is bus runs per period

same optimal fleet size of weekday off-peak periods. This differential cost indicates that the available bus fleet need not be fully utilized during a particular period, but to provide just enough service in order to minimize the bus operating cost. However, for the efficient system operation, most schedule periods should fully utilize available revenue-producing bus vehicles as shown in the column of the differential costs. All schedule periods other than the schedule period 3, differential costs are zeros indicating the maximum service frequencies are fully utilized.

Computation of Service Frequency

This application aims at two related objectives. The first is to compute the service frequency required during each schedule period for the new bus route. This service frequency computation is performed provided that the route is feasible based on transit cost reductions. The second is to expand this computation to include the derivation of service frequencies for existing bus routes.

The transit operations model structured in the linear programming problem is capable of analyzing the system cost effects of increasing or reducing the service frequency along a well defined route.

The needs of this application arise when bus passenger demand patterns have varied in such a way that will necessitate a change of headway. This analysis also applies when the transit planner expects either a change of travel paths between major activity centers, or a change of transit operating speed on major street links due to the modification of traffic operational characteristics. For example, provision of exclusive bus lanes for the expedition of bus operations, relocation of bus stops and general traffic engineering improvements for circulation, i.e. signal progression, street widening and installation of one way streets, have pronounced effects on bus transit operations.[4] Improvements such as these may affect costs for both the transit operator and the passenger.

When the transit planner has to deal with these situations, he has the planning alternatives of service increase, service reduction or total service abandonment. Before any alternate is selected as a solution, the transit planner has to analyze the potential cost impact of different alternatives. Furthermore, the incremental cost of service reduction or expansion must be known so that the selection of a solution will provide all necessary information on potential cost savings and the required service amounts.

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[4]For example, over 800 buses bypass congestion on New Jersey I-495 near Lincoln Tunnel via the Exclusive Bus Lane during three morning peak hours. The Exclusive Bus Lane was implemented in December, 1971 under the Urban Corridor Demonstration Program and saves 15 minutes per person on the average totaling approximately 2 million passenger-hours annually. The Exclusive Bus Lane also attracted additional 2,300 daily morning peak-period riders representing a 6 percent increase in 1971. For further information, see Goodman (88).

In order to apply the transit model for the computation of service frequency and associated transit operation cost, data must be collected on changes of transit demand and transit operational characteristics. Such data is usually collected by means of a transit survey and consists of passenger Origin and Destination information, bus speed and delay data and travel paths among major nodes.

The Origin and Destination information is usually collected over a period of a week and then further broken down into individual schedule periods such as the weekday morning peak period, off-peak period etc., to represent significant passenger demand variations over time. The speed and delay data is collected in the form of travel speeds on street links and delays caused by traffic signals and congestion.

If the problem is to examine the cost impact of changing service frequency on the bus route, the necessary data such as demands, travel time, operating cost, load factor, monetary value of passenger time and fare are entered into the model in order to compute the optimal transit operation cost for each state of service frequency and related passenger and bus flows. Next, if the problem is to examine the cost impact of changes in existing service on the total system, the input data to the model must be modified by considering the existing service frequency of the route under investigation.

Once the transit operation cost for different frequency increments is computed, the difference between the optimal cost and sub-optimal cost is computed. The percentage differences of these two costs are calculated by dividing the cost difference by the optimal transit operation cost as shown in Table 7. The result is a product of the first phase model and does not include the bus ownership costs for the provision of service frequency. However, for relative cost comparison, the result is significant with an average 27.5 percent cost difference for all periods and the highest, a 77.5 percent difference for Saturday peak periods. More specifically, the implementation of the optimal service frequency will reduce the transit operation cost by 27.5 percent on the average for all periods.

The first column of Table 7 refers to the schedule period for which a demand profile and the existing transit service are known. The second column represents the transit operation cost, while the optimal service frequency for the same schedule period is shown in column 3. Column 4 offers the maximum transit operation cost while column 5 shows the service frequency which causes the maximum transit operation costs within the range of available frequency states. The zero frequencies derived indicate that a lack of service frequency increases the transit operation cost by increasing passenger costs. For example, during A.M. peak periods, minimum cost is incurred with service frequency of 50, while without any service, the cost is increased by 60.9 percent

TABLE 7

COST COMPARISON 4

MINIMUM TRANSIT OPERATION COST
VERSUS
MAXIMUM TRANSIT OPERATION COST

No.	Schedule Period	Minimum Transit Operation Cost	Service Frequency For Min. Cost	Maximum Transit Operation Cost	Service Frequency For Max. Cost	Cost Ratio	Percent Dfference
1	Sun.	1,880	150	2,287	0	1.217	21.7%
2	Sat. Off	974	150	1,192	0	1.224	22.4%
3	Week Off	4,974	100	6,190	0	1.245	24.5%
4	Sat. Peak	129	50	229	0	1.775	77.5%
5	A.M. Peak	905	50	1,456	0	1.609	60.9%
6	P.M. Peak	1,495	50	1,847	0	1.236	23.6%
	Annual Cost	10,357		13,201		1.275	27.5%

Note: All costs are in Thousand Dollars
Frequency is bus runs per period

of the minimum cost. The optimal service frequencies derived here are based on system effects of new service on the whole system, however, they do not take fleet size into account. In other words, the fleet size does not impose constraining conditions upon the computation of these service frequencies. For service frequency constrained by fleet size, the results of the joint two phase transit model should be used.

Feasibility of a New Bus Route

Once the optimal transit operation cost is determined for each frequency state over the entire range of schedule periods, a bus fleet size cost matrix can be developed for the purpose of computing the fleet size and service frequency that produce the minimum annual transit planning cost.

As realized from the transit operation cost analysis, a saving of transit operation cost can be made by increasing the service frequency. However, an excessive increase of frequency will incur very high cost because of the increase of the bus operating cost. Therefore, the increase of service frequency should be made just enough in order to optimize cost savings.

A given fleet size can make only a limited number of bus runs during a specific period. Consequently, in order to increase service frequency, fleet size must be increased. If fleet size is increased, the fleet is going to remain in the system continuously. Therefore, the cost saving made from the increase of service frequency and new additional cost from the increase of fleet size should be compared with each other. This comparison is to determine whether to increase service frequency and, if a route addition is economically feasible, then to determine the required amount of service and the associated fleet size.

Based on service requirements and total transit planning cost, the optimal transit system configuration is arrived at using the transit model. As an optimal solution, the transit model may generate zero service frequency and zero fleet size on the proposed bus route for all schedule periods. For example, the transit service provided by the existing system without a new route may incur less cost to society than a new system with an additional new route.

This clearly indicates that the new proposed route under investigation is not feasible because the cost saving cannot justify the additional cost for the new route. The solution from the model considers all possible interrelations between the existing system and the new system as well as among schedule periods within each system. Therefore, the solution of the model can be used as an objective basis for evaluating the feasibility of the new proposed route.

Computation of Fleet Size

The problem of computing benefits of different fleet sizes appears in the model as the aggregation of differences between the benefits and costs generated by variations of fleet size. The principal concern of the transit planner in determining the optimal fleet size is to decide how many buses to retain during what schedule period and by what type of ownership, i.e. rent or own. Another problem occurs when the transit planner has to evaluate the cost impact of different fleet sizes either for the capital improvement of existing route or for justification of government subsidy for the addition of new service.

The answer to the former problem is particularly important in a situation where the passenger demand fluctuates greatly over different periods. In this case, the required fleet size should be adjusted accordingly. The latter problem also presents a complex question of how the optimal fleet size should be determined with regard to available funds, user requirements and existing service conditions.

In computing the fleet size, the basic data for the first phase model should be collected as discussed in the section of computation of service frequency. In addition to this data, fleet size increment, maximum range of increments, vehicle ownership cost, bus runs per period and schedule period weighting factors are entered into the second phase transit planning model. The model then sets the possible range of service frequencies from which the optimal service frequency is derived. The model compares systematically the cost savings of the optimal fleet size which provides, in turn, the optimal service frequency, by means of computerized dynamic programming routine.

The results of the model show varying fleet sizes during different schedule periods and the optimal service frequency that should be provided by each fleet size. Consequently, the transit planner can obtain fleet sizes during both individual and overall schedule periods, and also total transit planning cost which can be utilized for the transit policy decision-making process.

Impact of Parameter Variations Upon Transit Policy

Many aspects of transit operation and planning are represented by the transit model which is composed of various transit parameters. Change or modification of one or more of these parameters can have a profound effect on the service frequency, fleet size and the total cost. These changes may be either system wide or confined to a specific transit route.

Transit parameters can be generally grouped into three categories: transit user oriented, transit operator oriented and the network system oriented. As user oriented parameters, minimum passenger demand, passenger origin and destination information, demand distribution over time and space, load factors and passenger time cost can be chosen. The transit operator oriented parameters of, operating cost, ownership cost, budget, weighting factors, existing service frequency, existing fleet size and fare structure can be altered to fit particular service condition. In addition, as a system oriented parameters, different values for link operating speed, delay, bus stop location and street network can be selected.

The transit model can compute the cost impacts of changes in the transit parameters if the change is coded and entered into the model. Based on these changed parameters, the model computes all controllable decision variables such as passenger flow and bus headway in such a way that the total transit planning cost can be minimized. Once the new optimum transit planning cost for one set of parameters is derived by the model, it can be computed with another cost which is based on different parameters.

Likewise, a series of cost computations can be made for varying assumptions in transit user requirements, service conditons and system characteristics. This computation enables the transit planner to evaluate effects of transit parameter variations upon the total cost and the system consigurations.

As an illustrative example, the model is tested for its sensitivity to the variation of bus ownership cost. The bus ownership cost of the original transit input data is replaced with a new ownership cost to form a second set of input data. Appendix E shows the results of this additional application of the model using the second set of the input data. As a new ownership cost, a figure which is much higher than the original one is used. The new high ownership cost is based on the assumption that the capital investment for the retainment of the bus fleet is much more valuable than

CALCULATION ======>

PATH MATRIX

STATE/STAGE	1	2	3	4	5
1	3	3	3	4	4
2	3	3	3	4	4
3	3	3	3	4	4
4	4	4	4	4	4
5	5	5	5	5	5
6	6	6	6	6	6
7	7	7	7	7	7
8	8	8	8	8	8
9	9	9	9	9	9

* * * * * * * * * * * BUS SYSTEM STUDY RESULTS * * * * * * * * * * *

SCHEDULE PERIOD	FLEET SIZE
SUN	10 BUSES
SAT OFF	10 BUSES
WEEK OFF	10 BUSES
SAT PEAK	10 BUSES
A.M. PEAK	15 BUSES
P.M. PEAK	15 BUSES

SCHEDULE PERIOD	SERVICE FREQUENCY
SUN	150 DISPATCHES
SAT OFF	150 DISPATCHES
WEEK OFF	100 DISPATCHES
SAT PEAK	50 DISPATCHES
A.M. PEAK	50 DISPATCHES
P.M. PEAK	50 DISPATCHES

OPTIMUM FLEET SIZE OF PROPOSED BUS ROUTE IS 15 BUSES

FIGURE 9 BUS TRANSIT POLICY DECISION PATH MATRIX the transit planning cost as discussed earlier in Chapter V. The high value of ownership cost is because of the fact that the transit planning cost includes the passenger time cost while the ownership cost does not.

The effect of the higher ownership cost is clearly indicated by the results of the model which recommends to drop the proposed bus route for an overall optimum transit system. The decision path taken for this case is different from the original transit policy path matrix shown in Figure 9.

Summary

In summary, the application of the two phase transit model to a specific bus transit study demonstrated that the model is useful in measuring cost impacts of bus transit system configurations. Specifically, the model can be used, within the present limitations of the model, in the evaluation of the optimum versus sub-optimum transit systems, computation of service frequency, feasibility of a new route, computation of fleet size and effects of parameter variations upon transit policy. The evaluation of computational results found that the new transit model is an effective aid in estimating quickly and efficiently, impacts of transit system improvements and revisions upon transit service to the community.

CHAPTER VII

CONCLUSIONS AND RESEARCH RECOMMENDATIONS

General Conclusions

A systems approach to the optimal design of a fixed route bus transit system has demonstrated its value and advantage as an analytical bus transit planning tool. This tool developed in the form of two phase bus transit model, is especially useful in analyzing a multitude of bus transit variables and their interactions. Various bus transit system alternatives and system effects of service modification also became apparent through the use of the model in a systematic investigation for determining the optimal transit system.

The formalization of bus transit problems in concise mathematical terms provides a better insight into the complexity of bus transit planning as well as a flexible and objective evaluative criterion for bus systems analysis. Based on this evaluative criterion, which includes such major components as bus operating cost, passenger cost, passenger revenue and bus ownership cost, an optimal transit system configuration is determined. The optimal transit system, then, provides valuable information as to required bus service frequency, fleet size and route configuration for an overall optimal system. In addition, the model provides impacts of bus transit parameter variations on the system performance.

The model developed in this research is operational within the limitations discussed elsewhere herein and with the present data source. An optimal bus transit system for the study corridor has been generated by using the transit model and found to be a significant improvement over the existing system. Chapter VI contains a detailed discussion of the application of the model.

The model was developed to investigate both the fixed nature of bus transit operation during one schedule period and the dynamic characteristics of transit improvements over entire schedule periods. Therefore, an analysis of the transit system using the two phase linear and dynamic programming model was helpful in determining the extension or curtailment of service, and the effective coordination of bus transit with other forms of transport in urban areas.

The model, which was programmed for a digital computer, made it technically and economically feasible to investigate complex urban transit problems. The results of modeling efforts also implied that major system components, i.e., the transit user, the operator and the transit system, can be integrated for a more realistic systems analysis.

Transit System Modeling Efforts

The study has been directed toward developing a transit model that can be used to analyze and determine the optimal planning of a fixed route bus transit system through the use of a systems approach. For this end, a number of efforts have been made to keep the analysis as practical and operational as possible. First, the complexity of the transit planning process and the time limitation involved with the planning of a transit system in a congested urban area requires the selection of efficient systems analysis tools for developing the model.

The systems tools applied in this reasearch are linear programming and dynamic programming techniques which pose logically consistent concepts as well as efficient computational routines. The use of these techniques keeps the formulation of the transit operation in concise and logical format while at the same time permits the analysis of systems effects both within the transit network and among schedule periods.

The second type of effort at keeping the transit model operational for a realistic problem is in the development of the decomposition concept of transit operations. The decomposition, the breakdown of transit operation and planning into smaller entities, is proved to be efficient for the identification and definition of the multitude of transit system configurations. The decomposition is carried out both in time and space.

The basic planning entities as broken down here, include the transit planning cycle of a week, schedule period, service increments and transit corridor design. The cycle of a week is a method devised to define the transit system on a continuous time scale, while the schedule period is to represent variations of the transit demand profile, service amounts, and the transit network properties within the cycle. The service increment is a flexible measure of the magnitude of service frequency and fleet size during each schedule period. The corridor design is the geographical division of an urban area to represent uniform traffic characteristics.

The third way in which attempts are made to keep the model building internally consistent is through the use of the concept of degrees of freedom for modifying major transit system components. The transit route, fleet size and bus headway have different degrees of freedom. These components are restricted in their variation according to the order of their importance and impacts upon transit service for the community.

The restriction of variations of major transit system components enables the in-depth and exhaustive analysis of one system component before the next component or combination of components are analyzed. The restriction also facilitates the two phase development of the transit model. The first phase is for the transit operation during a single schedule period while the second phase model is directed toward the transit planning during multiple schedule periods by combining the analysis done in the first phase.

The fourth effort is directed toward the automation of transit planning techniques to provide quick and effective method for comprehensive transit system investigation. For this purpose, considerable efforts have been made to develop automatic computer routines for input data verification, data generation for the first phase transit operations model, and flexible data conversion for the use of available computer package programs.

Limitations of the Current Transit Model

In spite of many aforementioned efforts for comprehensive and operational model building, a number of assumptions are made to facilitate the study within the limits of a current data source and the time constraint of the research. Although some of the assumptions can be readily checked, others are much more difficult and require many years of research to obtain completely satisfactory results. The following list suggests the basic areas of limitations.

Fixed Route. With the concentration of regular bus transit demands and the limited street network suitable for bus routes in urban areas, it is most likely that regularly scheduled bus service should be on fixed routes. However, the transit planner may have ample reason to test variable route configurations. In this respect, the model is limited because its structure is based on the assumption that the revision of the route has the least freedom of change. It is possible to test a route with the model if the route is fixed during the planning cycle.

The Transit Planning Cycle. The predicted Origin and Destination information during various schedule periods of a cycle depends on the 24-hour average passenger demands which are derived from the census data. This method does not fully consider the fact that trip characteristics during each schedule period can be independent of the average 24-hour volume. It is also possible that transit demand may have monthly and seasonal variation in the area where recreational trip occupies an important portion of the total trip. With the use of a planning cycle, it is possible that the seasonal variation cannot be fully considered.

Load Factor and Demand Elasticity. The relationship between the transit service provided and the passenger demand realized is assumed to be a convex, non-linear function. This demand elasticity can be empirically derived. However, the accurate functional relationship between bus runs and passengers should be made based on trip purposes and trip makers. With the improvement of service, it is obvious that more passengers would be drawn to bus transit, but the load factors for different levels of service and their limits would need to be the subject of further research. The whole subject of transit demand elasticity is very important for balanced transit planning and it deserves an in-depth, long range analysis.

Costing. The monetary value of passenger time can be a subject of much speculation. In reality, the time values of walking, waiting and riding are somewhat different. The walking and waiting time may have higher value than that of riding time. In addition, the passenger waiting time is not considered as a separate cost item in the model. The present transit model can be expanded to include waiting time based on an average waiting time by considering a uniform bus headway and uniform passenger arrivals for a given service frequency. It is also possible that link operating time and cost may vary over time. In this respect, the present model cannot be readily applicable to accurate cost accounting. In addition, weighting factors for bus ownership cost need more rigorous investigation.

Analytical Tool. The passenger and bus flows in the resulting optimal system are based on the minimum cost path and do not preclude the possibility that individual passenger route preferences can be different from the theoretical minimum path. In addition, the structure of the first phase linear programming model and the second phase dynamic programming model is limited to the assumptions and constraints implicit in the techniques.

Implications of Results to the Study Objective

The results of this study have direct bearing on a number of transit planning problems for an efficient public transportation system in a congested urban area. Foremost in these implications is the development of a two phase transit model to meet the needs of the public transportation planning agency. The bus transit system modeling philosophy adopted in this research reveals that a model can represent the complex relationships among the multitude of bus system variables and parameters.

Particularly, this research demonstrated the value of systems techniques such as linear and dynamic programming tools in representing major bus transit components. However, some additional refinements of the methodology for the application of the systems techniques to large scale implementation may be necessary to make such an application both more practical and profitable. Derivation of the optimal transit operation during a given period and transit planning for overall periods requires analysis and investigation of a number of factors affecting bus transit performance and cost. However, because of interactive relationships among these factors affecting revenue, passenger cost and system operating and ownership cost, it is not adequate to analyze individual factors, taken one at a time.

Furthermore, a great number of bus transit system alternatives arise from variations of transit variables such as route, service frequency, fleet size and other operating policies. Accordingly, a systematic approach toward an analysis of the overall transit system will provide better insight into the complexity of bus transit planning in congested urban areas.

The primary purpose of this study is to develop an analytical technique to approach the transit planning problem from a systems viewpoint. The systematic approach aims at a better understanding of the complex interactions of transit system elements and to answer specific bus transit planning questions as:

1. Can addition or deletion of a bus route provide more feasible solution for an optimal system?

2. What is the optimum headway on the route being tested, given the existing bus routes and service?

3. What is the fleet size that offers maximum cost savings on the route, given the existing transit service, bus operating and ownership cost and other transit operating policies?

4. What are the bus and passenger flows that provide minimum transit operation cost?

5. What is the total operating cost, passenger revenue and passenger cost?

6. How does the change of network characteristics affect total cost?

7. How does the change of bus headway and fleet size affect total cost?

8. How does the bus ownership cost affect transit system configuration?

9. What will happen to the performance of the system when parameters of transit system vary?

10. When should the transit system provide more service and bus fleet?

The second implication of the study is the dynamic response of the transit model to fluctuations of various inputs to the transit system. The time varying and interrelated inputs are compiled from the basic transit system characteristics identified by the transit trip maker, the trip and the transit service. The dynamic procedures built into the model facilitate analysis of the transit system according to the transit system effectiveness derived as total transit operation and planning cost. Introduction of schedule periods, service frequency and fleet size states into the model make it possible to generate dynamic and plausible transit system configurations.

The third implication of the study has to do with the system effects of transit service. The solution by the model of the single period transit operation can be interpreted as a transit network equilibrium, i.e., as an assignment of passengers and buses over minimum path taking into account the capacity constraints of the network. This assignment considers existing transit service as well as proposed service on various routes. The solution also identifies system effects of one part of the system on other parts of the system. In addition, the progressive aggregation of single period transit operation over all schedule periods can be interpreted as the investigation of effects of transit system modification during one period on other periods. The solution of the model strongly indicates benefits obtainable from the analysis of transit system effects in time and space.

The fourth implication is the fact that even though the transit system analysis by the model is primarily concerned with one single route at a time, it can be also used to analyze the whole transit system. The application of the model in this regard indicates that an in-depth analysis of one route at a time can be more efficient and practical than a concurrent analysis of multiple routes.

The last implication of the modeling effort comes from the integration of major transit system components, i.e., the transit user, the operator and the transit system. Traditionally, the transit system analysis has been confined to the transit system network and the operator. For this reason, the model incorporates passenger costs with a built-in weighting factor and a parameter of passenger time into the derivation of the total transit planning cost. The passenger cost feature which includes time spent for walking, riding and transferring, allows the transit planner to weight major transit cost items flexibly according to transit policies. Thus, the value of passenger time can be adjusted according to prevalent local conditions. However, the equal treatment of passenger time cost with bus operating and ownership costs may make the passenger costs a major cost item in comparison with passenger revenue, bus operating cost and ownership cost. In a tight money market situation that requires more emphasis on available capital, the result of the model implies that passenger time value should be discounted, so that the bus operating and ownership cost can be more sensitive to the optimal transit policy decision.

The transit service improvement on line-haul is usually offset by the passenger inconvenience to get to the service and the waiting time for the service. This suggests that major efforts should be made to analyze the residential collection and the downtown distribution, more specifically, passenger time spent for walking, waiting and transferring. In order to provide for future refinement, the transit model has a structural capability for integrating bus passengers' walking, waiting and transferring on an average basis.

Future Research

A joint linear and dynamic programming transit model has been developed specifically to fit the context of a bus transit system in an urban area. However, it is emphasized that the model is not a finished product, but rather a prototype which provides guidelines and insight into solutions of complex urban transit problems. The study is basically experimental and has left many relevant questions and hypotheses unanswered. Additional research is needed in three related areas. They are (1) the development of an efficient data collection mechanism, (2) the derivation of validated bus transit parameters, and (3) the refinement of computational procedures.

Data Collection. The successful planning of a bus transit system depends mainly on the availability of data. Accurate passenger demands and Origin and Destination information are especially vital. The bus headway and transit route configurations must be constantly analyzed and modified to meet greatly changing conditions and varying needs of transit passengers. The amount of work involved in traffic counts, balance and projection to the future is considerable. The collected data and counts also must be reduced and summarized for their efficient use in the transit system analysis. A high speed automatic data collection method should be developed to facilitate continuous transit system analysis within time and budget limits of the transit planning agency.

Bus Transit Parameters. The complex transit system in urban areas is represented by a number of parameters and variables. Without validated parameters, the results may not be reliable. Especially transit demand elasticity which refers to the relationship between anticipated passenger demand and the provided transit service deserves further research. The derivation of transit demand elasticity is a difficult task because anticipated passenger demand is a function of many social and economic variables such as trip purpose, time, car ownership, income, sex and relative travel time ratio just to name a few. The transit demand elasticity and load factors approximated from it may be synthesized from comprehensive existing transit data. However, since the new ridership, drawn by better service may not have the same characteristics as existing transit patronage, there is a need for developing an advanced method of forecasting bus transit usage which can integrate the most relevant socio-economic factors affecting both existing and new bus ridership. In conjunction with parameters, the cost weighting factors introduced in the model should be more accurately validated by using detailed transit cost models.

Computational Procedures. Once a viable data base is obtained, it is possible to analyze an actual bus transit system network in an urban area in order to determine the optimal transit system configuration. However, the size of problem is relatively large even for a small network due to the great number of Origin and

Destination pairs and potential trip paths. The analysis and computation of the model is very time consuming and inefficient, especially for the first phase transit model structured in linear programming. Accordingly, a more efficient computational algorithm should be developed to make the application of the model to a larger area reasonably quick and efficient.

Summation

It is the intent of this research to apply the systems approach to the description and investigation of the bus transit system in an urban area. Based on this approach, the dynamic interactions of urban bus transit system components can be efficiently analyzed by using the two phase model.

In this research, two innovative concepts are incorporated into the bus transit model. The first notion is that a transit system should be analyzed from the total systems viewpoint. The second is that complex transit system improvements can be systematically investigated by an analytical model such as is suggested here. The application of these concepts in the context of urban transit systems analysis is shown to be both rewarding and educational.

In summation, a continued study and refinement of the model in the areas of efficient collection of transit data, derivation of valid transit parameters and the improvement of computational procedures can raise the efficiency of the operational model of urban transit system planning. The two phase transit model developed in this research has demonstrated its utility as a tool for dynamic transit improvement planning for the simplified study area. Using this transit model, the public transportation planner and the transit operator could provide much greater efficient and effective transit system in congested urban areas.

APPENDIX A

SUMMARY OF NOTATION

A	=	Chain-link incidence matrix
AOWC	=	Annual ownership cost
B^n	=	Budget limit during nth period for bus operation
C	=	Column vector of link bus capacity
C'	=	Column Vector of physical link capacity
$D(f^n, f^{n-1})$	=	Decision cost to transform fleet size from f^{n-1} to f^n
DL	=	Layover Time
DT	=	Turn Around Time
E	=	Incidence Number
F	=	Fare for dth demand
FN^n	=	Fleet size during nth schedule period n
H	=	Headway
I	=	Rate of return
L_u	=	Load factor for "u" level of service
N	=	Maximum bus passenger origin and destination number
N_c	=	Maximum chain number
N_1'	=	Maximum link number
N_s	=	Maximum schedule period number
OC	=	Operating cost of link
P^n	=	Number of hours in nth schedule period
PR	=	Purchase price
ROC	=	Route operating cost

S	=	Net Salvage value
SB^n	=	Schedule budget for nth period
T	=	Row vector of link running time
U^n	=	Transit operation cost for nth period
$\overset{*}{U}{}^n$	=	Optimal transit operation cost for nth period
U'	=	Upper limit of load factor
V^n	=	Total transit planning cost for period $\geq$ n
VN	=	Number of bus purchased
W	=	Row vector of bus passenger cost
X	=	Decision variables for assigned passengers
c	=	Chain number
d	=	Demand number
k	=	Number of fleet size increment
l'	=	Link number
m	=	Number of frequency increments
n	=	Variable schedule period
r	=	Average passenger demand
t	=	Estimated service life of bus vehicle
u	=	Level of service
v	=	Schedule speed
a	=	Weighting factor of ownership cost
b	=	Weighting factor of route operating cost
e	=	Weighting factor of ownership cost
Δ	=	Unit increment of frequency
f	=	Passenger distribution factor
θ	=	Unit increment of fleet size

APPENDIX B

REVIEW OF RELATED LITERATURE

Introduction

In order to acquire an understanding of current bus transit system planning techniques and research works, a substantial effort has been devoted to examining previous studies and then applications to the planning of a bus transit system. The previous research efforts can be generally categorized into studies of socio-economic impact, operational policy and bus hardware innovations. Within these broad categories, a review of the relevant literature and its relationship to this study was made with respect to components of a bus transit system, relevant transit factors, bus transit problem formulation and the selection of a solution method.

The components of a transit system are those attributes that characterize the transit service. They are the transit user, the system operator and the system itself. The relevant factors to be considered in conjunction with a transit study are those measures that affect the transit service and should be considered for the solution of the transit problem. Some of the more relevant factors are operating speed, delay and costs.

Furthermore, the transit system can be viewed from many different points. For example, the system can be analyzed from the bus management point of view while it is also possible to examine the service from the user's point of view. Accordingly, the transit system can be formulated in a variety of ways to identify problems. The solution method refers to the approaches to the problem, i.e. heuristic or mathematical programming techniques employed for the solution of the formulated transit problem. The review of previous study is made according to (1) transit system components (2) relevant factors (3) formalizing of problems and (4) solution methods.

Transit System Components

Much research has been conducted for the investigation of transit system components. This research has been concentrated especially in the area of the transit model building, computer simulation, scheduling, inventory analysis and operating cost analysis.

In addition, the transit user and operator requirements were also investigated. The questions related to user benefit which should be also reviewed from a management point of view include network flow and structure, fare structure, vehicle size, fleet size, manpower assignment, terminal requirements and location, and bus priority consideration.

In the area of network flow many researchers have made theoretical contributions. The flows on network links are, however, determined by traffic assignment techniques. The simplest assignment is "all or nothing" which assigns demand according to a minimum time path (or cost path) between origin and destination. Another variation of this is the assignment by recomputing travel time after considering capacity constraints of links. The assignment is continued until system reaches equilibrium. Another method of determining the traffic flow is by linear programming methods which attempt to minimize overall travel time subject to resource constraints such as equipment and manpower. Manheim (34) and Tomlins (44) are concerned with determining network flows in equilibrium through linear programming. Synthesis of networks were the concern of many researchers such as Carter and Stowers (6), Quandt (39), Hershdorfer (90), Hay, Morlok and Charnes (24) and Ocha-Rossa (103). The practicality of the models manifested especially by Hershdorfer, who developed a model to design urban system networks by determining optimal link additions and directionality of traffic flow, and Hay, Morlok and Charnes (24) whose model determined the optimal mix of rapid transit and highway capacity.

The user benefits have been analyzed broadly in two categories. The first category measures individual travel properties. These properties include trip purpose, fare and level of service, passenger preference between departure oriented or arrival oriented (123), and duration of trip as was considered in the computerized school bus model by Tracz and Norman (45). The second category tries to aggregate the user satisfaction in terms of total travel time, total delay and waiting time, and demand elasticity as in the case of Webster (120).

The user requirements usually impose such constraints as maximum walking distance to a bus stop, maximum waiting time, and clear information on scheduling. Maximum walking distance depends on route network, bus stop organization and user characteristics. Peterson (37), investigated the average walking distance by people in the Washington, D.C. residential area. By considering car ownership and socio-economic status, walking patterns of people from their home to a bus stop was analyzed and statistically computed to get the mean walking distance, standard deviation and standard error. The source of data was an Origin-Destination questionnaire completed by selected bus riders in Washington, D.C.

Maximum waiting time is related to bus headway and to the vacancy of bus seats. The headway is, in turn, directly related to number of bus dispatches over the route network. The cost of operating a certain size of bus fleet is primarily due to the number of required dispatches. This is the reason why the scheduling problem is one of the most relevant factors in bus transit system operation.

Relevant Factors

The factors affecting transit service are mainly operating speed, delay, headway, cost and traffic engineering features. All these factors are closely related to bus transit scheduling. Therefore, scheduling is a sensitive element of transit system improvement. Scheduling refers to such functions as selecting vehicle headways, constructing time tables and dispatching vehicles for trips. Scheduling is a complex and time consuming task. A high speed ground transportation simulation by Crane (75) and the airline simulation projects by Kingsley (28) concern the quantitative measures of scheduling in terms of cost and utility performance.

The determination of the required vehicle inventory for implementing a fixed timetable was given much attention by transportaon researchers. Seshagiri, et. al. (40) studied bus schedules for large bus transport network, and Lines, Lampkin and Saalmans (30) for a municipal bus undertaking were concerned with computing minimum vehicle requirements as part of overall schedule determinations. In addition, Simpson (112) has included minimum fleet size for an air-bus system. In the context of railroad systems, White and Wrathall (121) dealt with a problem of scheduling the actual movement of all cars.

In the context of real world bus system, Tracz and Norman (45), have developed a computerized approach for route design, vehicle assignment and time table development for a school bus system. Others such as Eliaas (81), and Lines, Lampkin and Saalmans (30) have directed their investigations to obtain a methodology for economic scheduling.

In a study by Lines, et al. (30) the travelling requirement of the public was defined by demand nodes and matrices for different days and different periods of each day. The problem was simplified by assuming that there is no short-term relationship between service and usage, consequently the income was assumed the same for all plans, and the differences in profit between different schemes were the differences in cost.

Further simplification was made with the approximation that the major bus transit operating cost consists of only bus crews, and so minimizing total travel time subject to a given crew strength is equivalent to minimizing total travel time subject to a fixed level of profit. The problem of choosing service frequencies was formulated as the minimization of the total travel time subject to the total fleet size. An heuristic algorithm was developed in order to produce the necessary route network. When routes and frequencies had been determined for each period, timetables were drawn up, and bus and crew schedules prepared.

Another element of bus transit system improvement closer to real world problem is a traffic engineering application to efficient and smooth bus system operation on existing street networks. The techniques considered usually include bus priority and traffic control, park and ride, bus stop location and access. Bus stop locations and lengths (70) were investigated in relation to safety and traffic flow. The advantages and disadvantages of bus stop locations i.e., near side, far side and mid-block were analyzed in reference to various bus and traffic movements. Besides the location of bus stops, the overall organization of stops into express and local stops for different service modes are important for system utility since each added stop generally decreases the average bus operating speed, increases delays for a majority of the passengers and causes traffic congestion. Little work has been done in this area. Black (3) was concerned with determining a break point on radial routes of rail transit to employ local trains between Central Business District and the breakpoint and express trains carrying through passengers non-stop from the breakpoint to Central Business District. The total cost consisting of equipment cost, construction cost and travel cost was expressed as a function of the location of the breakpoint from Central Business District. The practice of bus stops for freeway operation has been reviewed by Homburger and et al. (60), and Rainville (108).

Determination of operating cost is another essential element of bus transit improvement. Operating cost is usually a function of route miles, route running time, required number of vehicles, vehicle-miles, vehicle-hours, layover time and efficiency of scheduling. These items are an integrated part of every transit improvement study in part or in combination. Studies done by Nemhauser (36), Ward (119), Devanney (77), and Lines, Lampkins and Saalmans (30) are directly concerned with this aspect of bus undertaking. In addition, user costs such as walking distance, waiting time, stop and delay and maximum speed cost are considered.

In the area of bus operation run-cutting, several attempts were made to computerize the assignment of crew and vehicle in bus transit. Elias (81) formulated this problem through methematical programming. Integer linear programming was used and the objective function was set up to include splitting of runs as decision variables. For even a simple route, it was discovered that the model is considerably beyond the ability of current integer programming algorithms. Accordingly, heuristic programming techniques were developed to simplify the problem.

Formalizing the Problems

The problems of urban bus transit operations are complex. Their complexity requires the use of many different methods to relate diverse system elements to the system objective. Previous researchers have placed emphasis on different aspects of transit system elements.

In the area of determining optimum bus service by developing optimal route, frequency, bus sizes and service mode, several efforts have been made. Webster (120) estimated the effect of London car commuters transferring to bus travel. The possibility of using several different sizes of buses were investigated by assuming that all commuters are transferring to alternate system of uniform size of bus. Such factors as passenger car unit equivalents of different size buses at intersections, passenger carrying capacity of street, vehicle travel speed as a function of traffic flow, total travel time and route density were considered to compute the cost to operators and the cost to passengers in terms of time and direct expenditures. The total travel time was calculated after considering the effects of bus stops and bus flows on traveling speed, and also the effect of route density and service interval on minimum waiting and walking times. All these calculations were based on the assumption that buses are running in uniform urban area and all figures are related to average journeys and not to a particular one. Therefore, it does not provide any information on the actual route location and timetable construction.

Another simple theoretical model of bus service was also concerned with a large uniform area. Holroyd (92) developed a method of finding the optimum bus routes and frequencies in a large uniform area with a grid system of routes and the same frequency of buses on each route. Formulae are derived to give the average times on the trip spent walking, waiting and riding buses in terms of the parameters of the model. The optimum route spacing and frequency minimizing the system objective such as the time cost of travel plus cost of providing bus service were calculated Mathematically.

An area of bus transit improvement that concerns researchers is the development of a method for analyzing bus transit system on a computer to determine the usefulness of an alternate system in comparison with the existing system configuration. Seshagiri, et al. (40) developed a method of analyzing a large transport network on a digital computer to improve the utilization of buses and the duty allocation for the crew without collecting extensive data. The approach to the problem solution was to minimize the sum of the fixed cost and the variable cost with the parameter values lying between the lower and upper bounds prescribed by the problem. The objective function to find its minimum was represented in terms of vacancies, distances between stops and the capacity of a bus. Mathematically, the objective function is the inverse of capacity minus vacancies multiplied by distance between stops and the summing through the range of all trips and stops. The objective function was minimized by perturbing the headway list, which in turn perturbs the arrival time of a bus at each stop, which then perturbs the vacancy, forming the objective function as an independent variable. In this study, a reduction of the number of trips during the non-peak hours was the prime objective. The optimum headway list was averaged together with the running time subject to various constraints.

The logical structure of a model directed toward bus system improvements can be expressed in an objective function and a set of constraints. The need for a carefully chosen objective function is evident since it is a measure of system optimization. The objective function should provide a good measure of service impacts and the various cost components. Objective functions formulated for optimization models differ in relevance to system criteria and their purpose. Some simple objective functions were structured to take account of only prespecified constraints imposed either by demand sides or supply sides. Another set of objective functions follow strictly economic outputs such as maximization of revenue or minimization of cost (3).

More elaborate formulations synthesize the level of service and costs such as combination of total travel time, total waiting time and delay, and system operation cost. The dynamic programming formulation by Devanney (77) and Ward (119) are good examples of this type of objective function. It is foreseen that more analytical efforts will be directed in the synthesis of cost elements and to its optimization.

Demand responsive and dual mode system have been paid much attention by many researchers (20) as a future transit system and this effort will no doubt help to develop a "real" time system to handle door to door demand. However, more research efforts are required in the area of planning the bus transit system being operated on fixed routes with high service frequency in congested urban area.

Solution Methods

Finally, it may be useful to review solution techniques in current use. The solution process for bus transit improvements may not be identical in all cases and may differ depending on the elements included and the special nature of the problem. The nature of bus transit improvements is quite complex and a wide variety of physical characteristics are encountered in practical problems. The complexity and the different structural characteristics of the problem clearly indicate the need for a variety of techniques to cope with the solution of the problems. The range of techniques include statistical decision theory, game theory, control theory, calculus of variations, mathematical programming, simulation, analytical approach and heuristic algorithm, etc. These techniques are being applied independently or combined during the optimization process. Black (3) used the analytical approach for passenger car dispatching policy and selection of service mode. Foulkes et al. (15) solved the sequencing of buses in a network with a set of linear simultaneous equations. Beckman et al. (54) determined the best freight schedule in a simple network with an analytic solution. Simulation and experimental methods have been used by Gunn (22), Howard and Eberhardt (13), Crane (75) and Kingsley (28). Heuristic algorithm have proved to be a powerful tool to handle a complex problem. Elias (81) developed heuristic programming for crew and vehicle assignment. Lines et al. (30) also employed this technique for municipal bus route construction. Gagnon (17) assigned passenger to flights based on heuristic procedures.

Mathematical programming has been widely used as a powerful optimization technique. Linear programming and variations of this technique have been used by many researchers. Manheim and Martin (34), Tomlins (44), Hay et al. (24), Hershdorfer (90) and Hartgen (89) are all good examples. Network flow theory (58) was also used by Simpson (112) for computerized schedule construction for an airline system. Dynamic programming techniques were applied by Devanney (77), Ward (119) and Young (123) for the solution of transit scheduling problems.

In the area of computer simulation, analysis of bus transit system by a series of computer programs developed for long-range public transit system planning, was conducted for the Washington, D.C. Transit System by Voorhees (114). One of the primary objectives of this study was to investigate the effectiveness of a transit system simulation through computer methods as a short-range planning tool. First, in order to develop the basic optimum bus route system, alternate systems were developed in succession based on routing criteria such as route simplicity, avoidance of loops and maintenance of existing cost structure, etc. An evaluation based on travel times, numbers of transfers and operating cost was then made. A revised set of special purpose routes was added to the basic optimum system to serve demands not covered by basic system. In fact, the optimum system thus developed is not a global optimum system, but provides the best system among alternates. After running times and the route structure were determined, scheduling was processed using the basic information such as maximum boarding-alighting counts and scheduling standards to calculate bus headways for each route.

In last few years, dynamic programming concepts started to be used by mass transit researchers such as Devanney (77), Ward (119) and Young (123) as an aid in multi-stage decision process toward overall optimal system operation. Dynamic programming is a technique to find a best solution among several feasible alternatives. Dynamic programming was first theorized by Bellman (52) whose book on the subject was published in 1957. Dynamic programming provides a systematic procedure for determining the combination of decisions which maximizes the objective. The dynamic programming problems can be basically divided into stages, with a policy decision required at each stage which has a number of states associated. The decision making at each stage transforms the current state into a state in the next stage. After dynamic programming was developed, many problem areas, such as control processes, inventory theory, and allocation, were approached by applying this sequential decision process for their optimization.

An initial application of dynamic programming was made by Devanney (77) to develop optimal one-way timetables for dispatching vehicles on a linear network. Ward (119) developed computer programs to implement this algorithm for different types of network configuration. As a criteria of optimality, passenger delay and system capacity were chosen, and the objective function to be minimized was expressed in terms of a weighted sum of passenger delay and system capacity. The decision times were predetermined arbitrally on the fixed time horizon, and optimal decision was sought for at at each stage among alternate decisions which were prespecified in order that the objective function incurred the minimum cost. This calculation was performed backward through the full range of decision stages based on the recursive relationship of dynamic programming. In order to facilitate the calculation of passenger delay, the distribution of passenger arrival was transformed into a function of time. Decision times were spaced at equal increments of passenger arrivals.

Young (123) was concerned with a method for developing efficient timetables for the operation of fixed schedule common-carrier passenger transportation systems. The timetable optimization is accomplished by maximizing an objective function consisting of three basic components, operating costs, revenues and traveler benefits. The method of optimizing a vehicle fleet timetable is based on successive use of a dynamic programming algorithm that computes a currently optimal schedule for a single vehicle. At each stage (stage was defined as a discrete time variable), the dispatch decision was made by maximizing profit over the destination node, and the service mode and network path to get there. The alternatives include a decision of remaining at the current node until the next decision stage. It was difficult to deal with the practical problem to get the optimal solution due to the multi-demensionality of the state variables.

In summary, various parts of transit systems have been studied in order to identify, formulate and analyze complex problems of urban transit service. The approach to the problem and the selection of the solution methodology should be considered in relevance to the problem size, computational facility and the transit planning objective.

APPENDIX C

FLOW CHARTS AND LISTINGS OF COMPUTER PROGRAMS

The basic process of computation as well as interactions among computer programs comprising the bus planning model are illustrated here to show how the optimum bus transit system is determined. The planning model is a joint linear and dynamic programming model consisting of four major computer programs which perform the necessary computations.

These programs are an input generator program (LPDGEN), a linear program (IBM MPS/360), a dynamic program (LEEDP) and a conversion program (LPTEST). A conversion program is developed to flexibly utilize available linear programming package programs which are based on different solution techniques such as the two phase method and the revised simplex method. The first program, the input generator, supplies input data to the package linear program according to the structure of the transit model described in Chapters IV and V.

The main purpose of the input generator (LPDGEN) is to mechanize the time consuming preparation of input data to the linear program. The generator is also used because the linear program requires a precise order of contraints and objective function. This program also converts the matrix notations of equations into regular linear equations through a series of multiplications of problem matrices. The function of this program is:

1. Perform program specification.

2. Read general linear programming structural data such as nature of objective function (minimization or maximization), number of constraints, number of variables, number of "less than or equal" constraints and number of "greater than" constraints.

3. Read basic bus transit system input in the order of number of passenger demand, number of chains connecting particular origins and destinations, number of links, load factor, chain-link incidence, monetary value of passenger's time, link travel time, link operating cost and bus fare.

4. Verify input data.

5. Generate service elasticity matrix.

6. Generate link service capacity matrix using incidence matrix and load factor.

7. Generate fleet size constraints.

8. Generate budget constraints.

9. Generate cost coefficients for objective function by combining passenger cost, bus system operating cost and revenue. The typical output of this program is shown in Appendix E. The flow chart of the program illustrates the logic of computation.

After the matrices of coefficients of the linear programming model are generated by (LPDGEN), the proper right-hand-sides[1] of equations are added to the input. The input, then, is converted according to

---

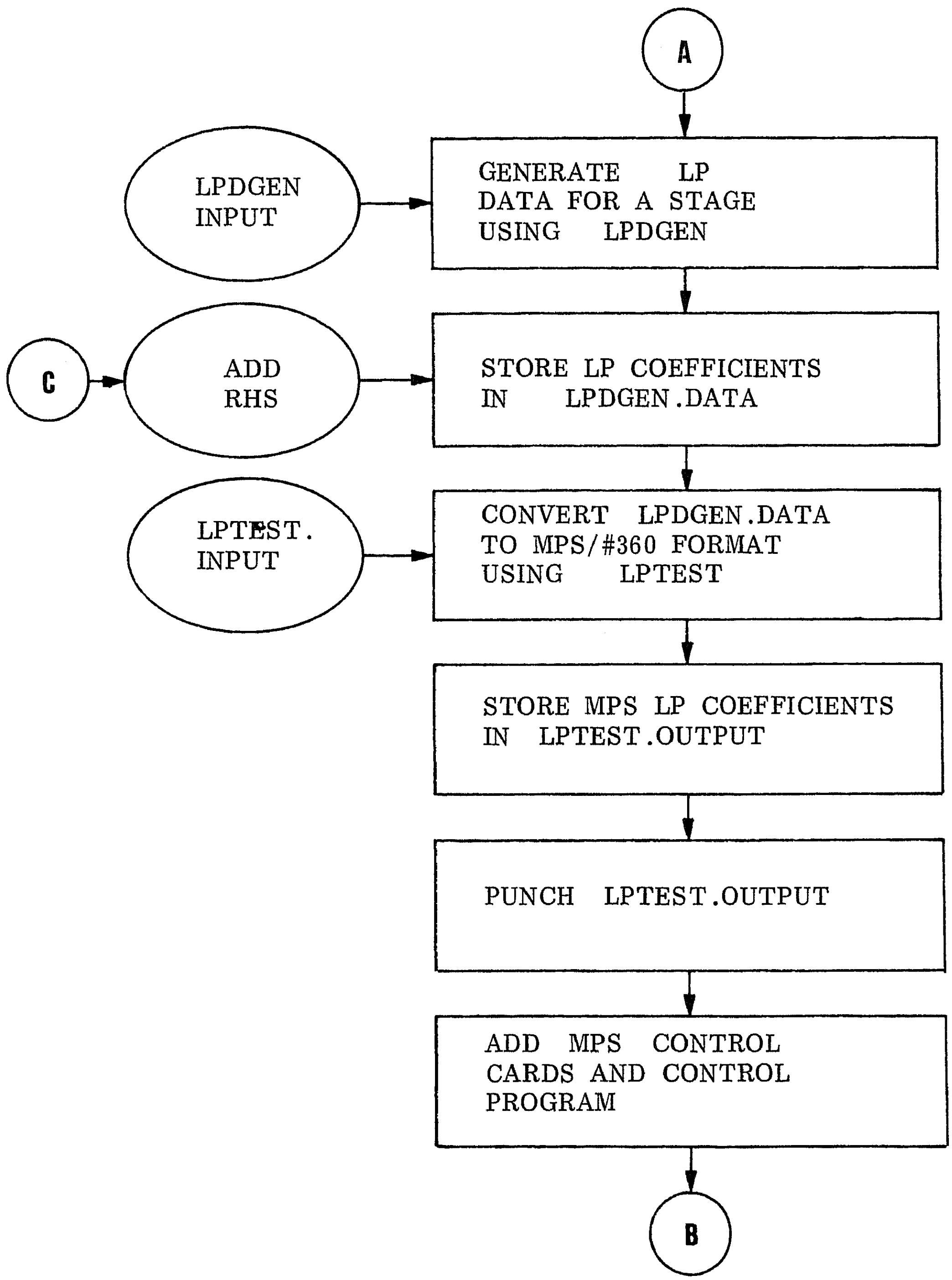
[1]The right-hand-sides will change over states for those links covered by the proposed route within the same stage. Also, right-hand-sides will vary over different stages because of change of service.

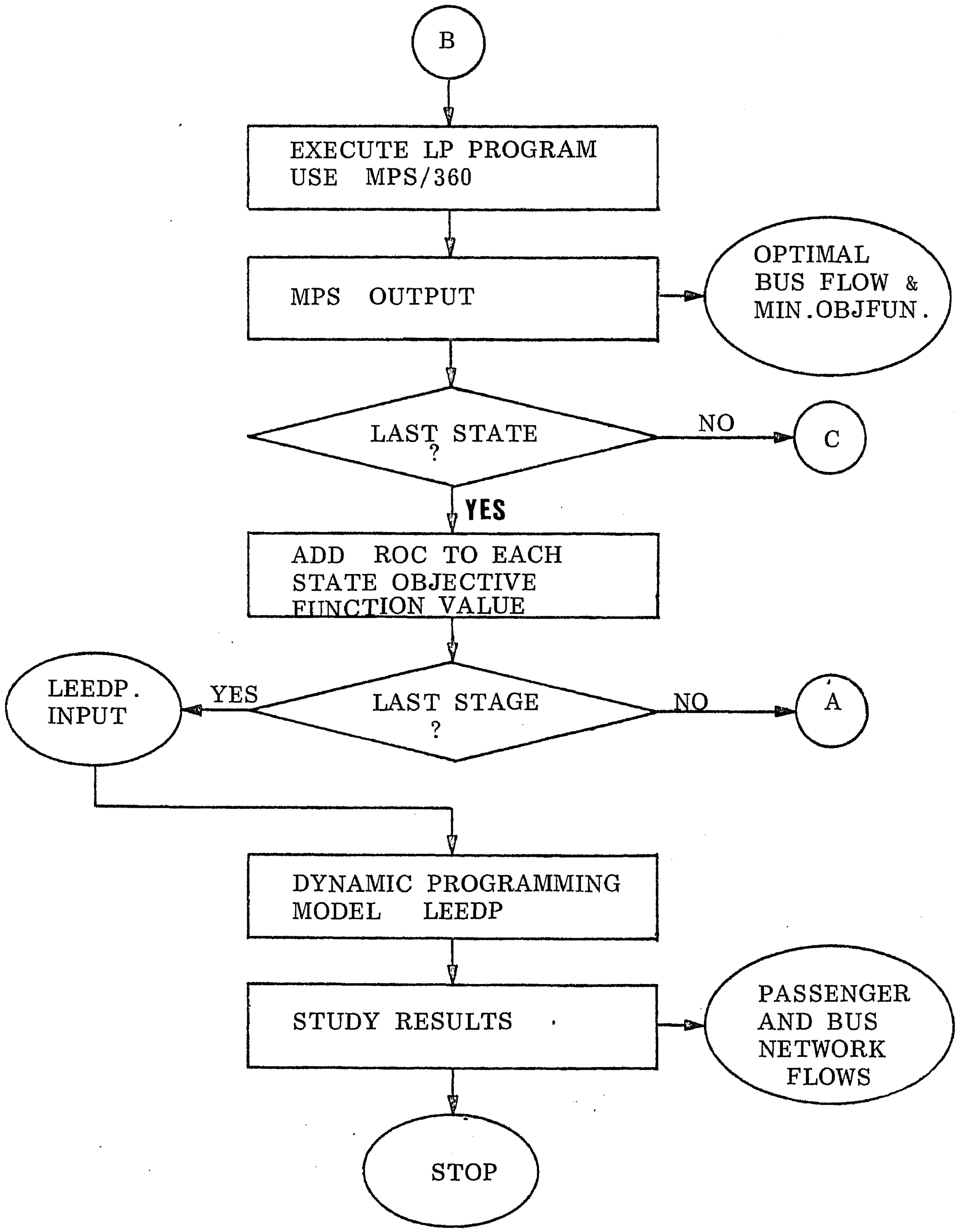
the specification of a particular package program. The actual computation of the sample test case was performed by using IBM/360 Mathematical Programming System at Princeton University accessed through Newark College of Engineering computer center. In order to make flexible use of the linear programming computer operation, computer inputs are also provided for other linear programming package programs, such package program as LINPRO developed by Dartmouth College in both Basic and FORTRAN IV language and SSLP of RCA.

The outputs from the IBM MPS/360 linear program are a job control language, a control program listing, and a summary of minor and major errors. Following these outputs, the optimum solutions and related information are produced. A sample output of linear programming is shown in Appendix E. The total elapsed time of a typical run of a problem with 39 constraints and 66 decision variables is 66 seconds. Once the linear programming run is finished, then the optimum passenger flow and associated bus fleet assignment on the street network are known. All basic information necessary for the dynamic programming phase is stored in the file name (LEEDP INPUT) for the execution of the dynamic programming. Input to the dynamic programming program is prepared after bus transit operation is optimized in both space and service quality for each schedule period and each service frequency. This optimal transit operation is represented by the objective function value which is minimized based on the simplex algorithm.

BUS TRANSIT PLANNING MODEL
FLOW-CHART FOR DATA FLOW FOR LP/DP MODEL

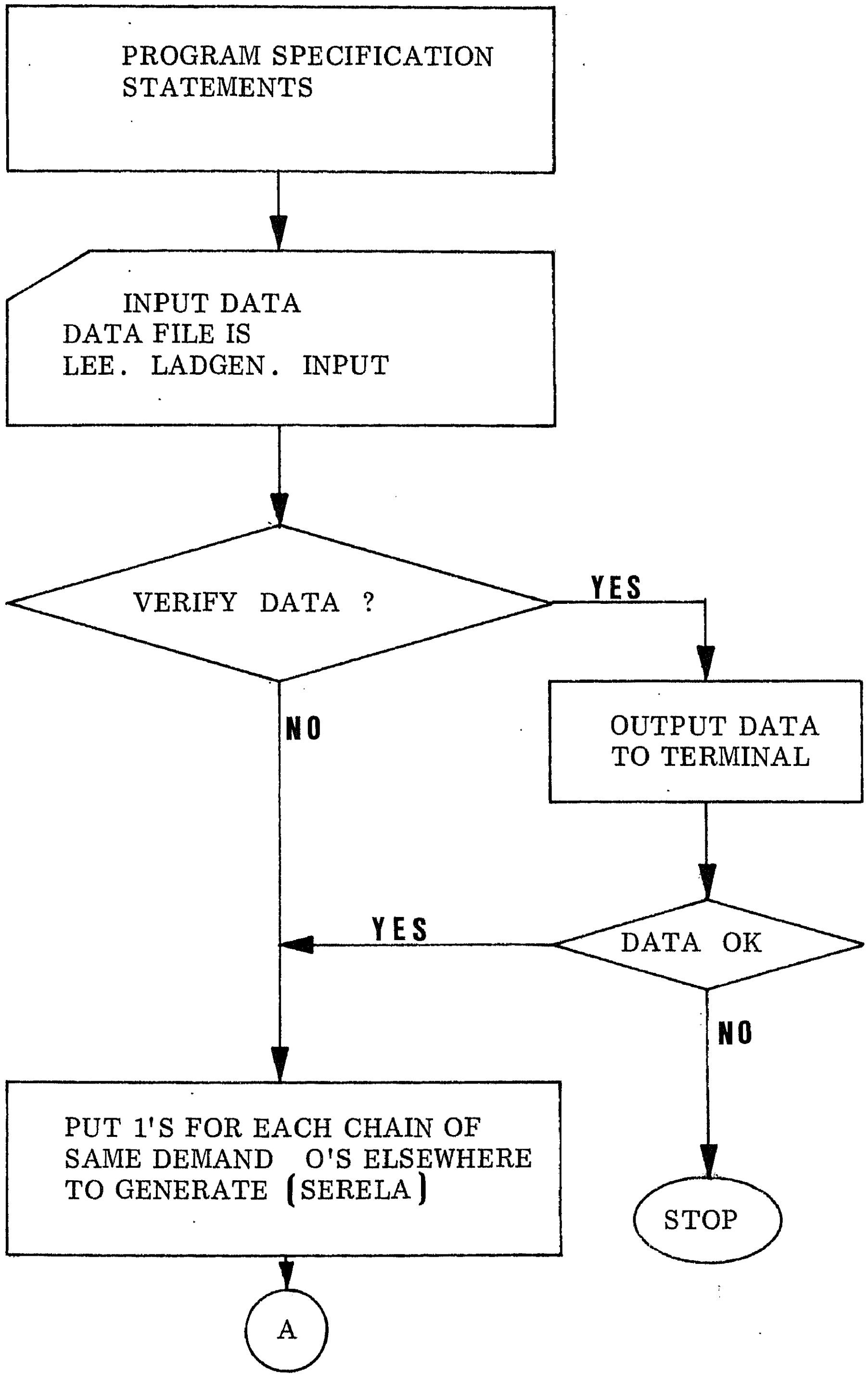
June, 1973

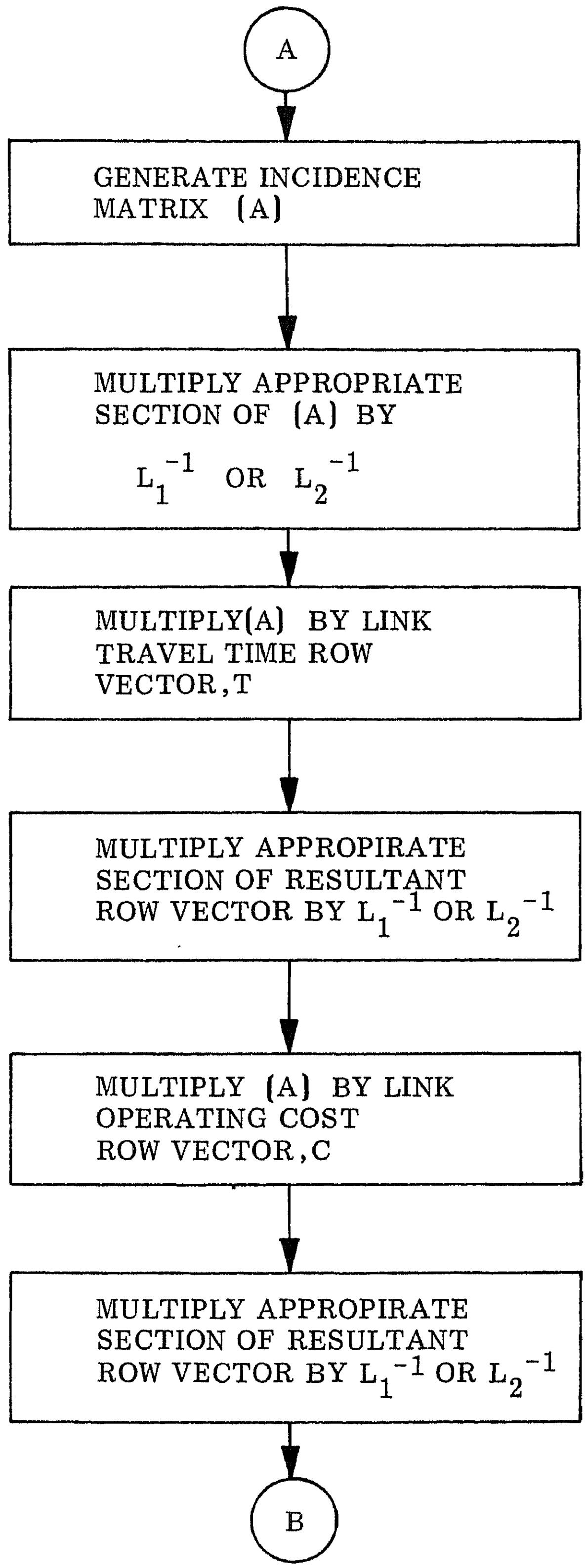


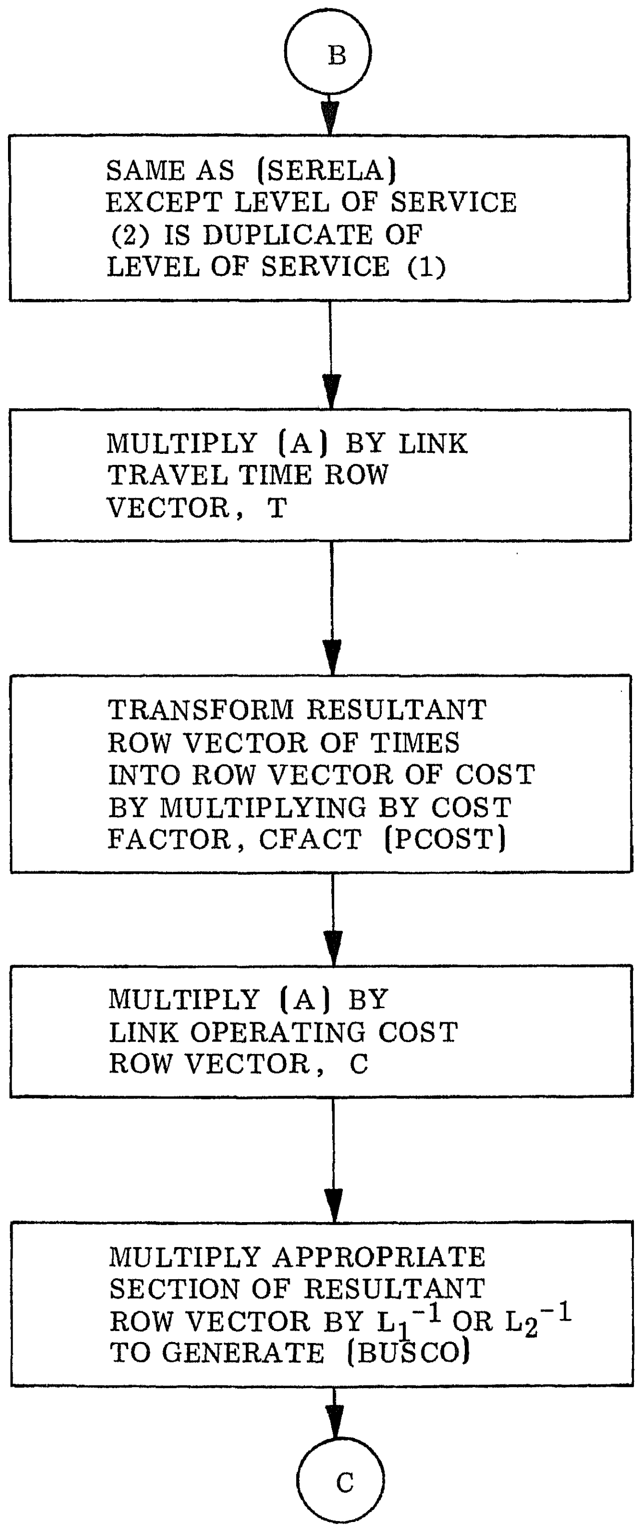

B
EXECUTE LP PROGRAM
USE MPS/360
MPS OUTPUT
OPTIMAL
BUS FLOW &
MIN.OBJFUN.
LAST STATE
?
NO
C
YES
ADD ROC TO EACH
STATE OBJECTIVE
FUNCTION VALUE
LEEDP.
INPUT
YES
LAST STAGE
?
NO
A
DYNAMIC PROGRAMMING
MODEL LEEDP
STUDY RESULTS
PASSENGER
AND BUS
NETWORK
FLOWS
STOP

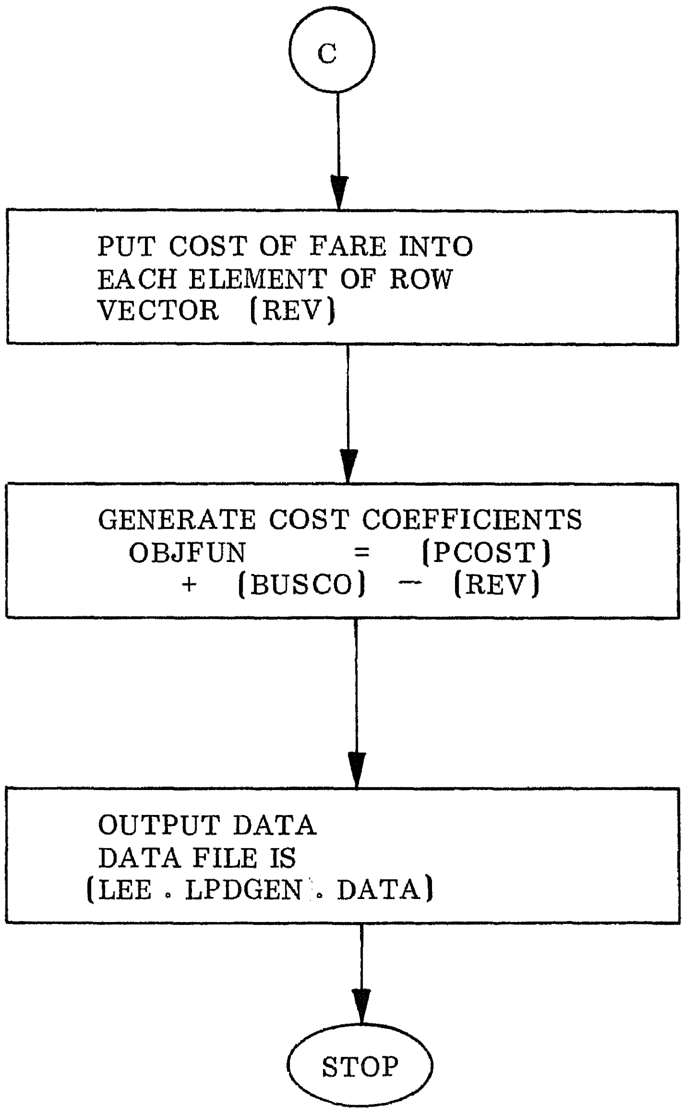
BUS TRANSIT PLANNING MODEL FLOW-CHART FOR
L.P. GENERATOR PROGRAM (LPDGEN)

June, 1973




A
GENERATE INCIDENCE MATRIX (A)
MULTIPLY APPROPRIATE SECTION OF (A) BY
L_1^{-1} OR L_2^{-1}
MULTIPLY(A) BY LINK TRAVEL TIME ROW VECTOR,T
MULTIPLY APPROPIRATE SECTION OF RESULTANT ROW VECTOR BY L_1^{-1} OR L_2^{-1}
MULTIPLY (A) BY LINK OPERATING COST ROW VECTOR,C
MULTIPLY APPROPIRATE SECTION OF RESULTANT ROW VECTOR BY L_1^{-1} OR L_2^{-1}
B

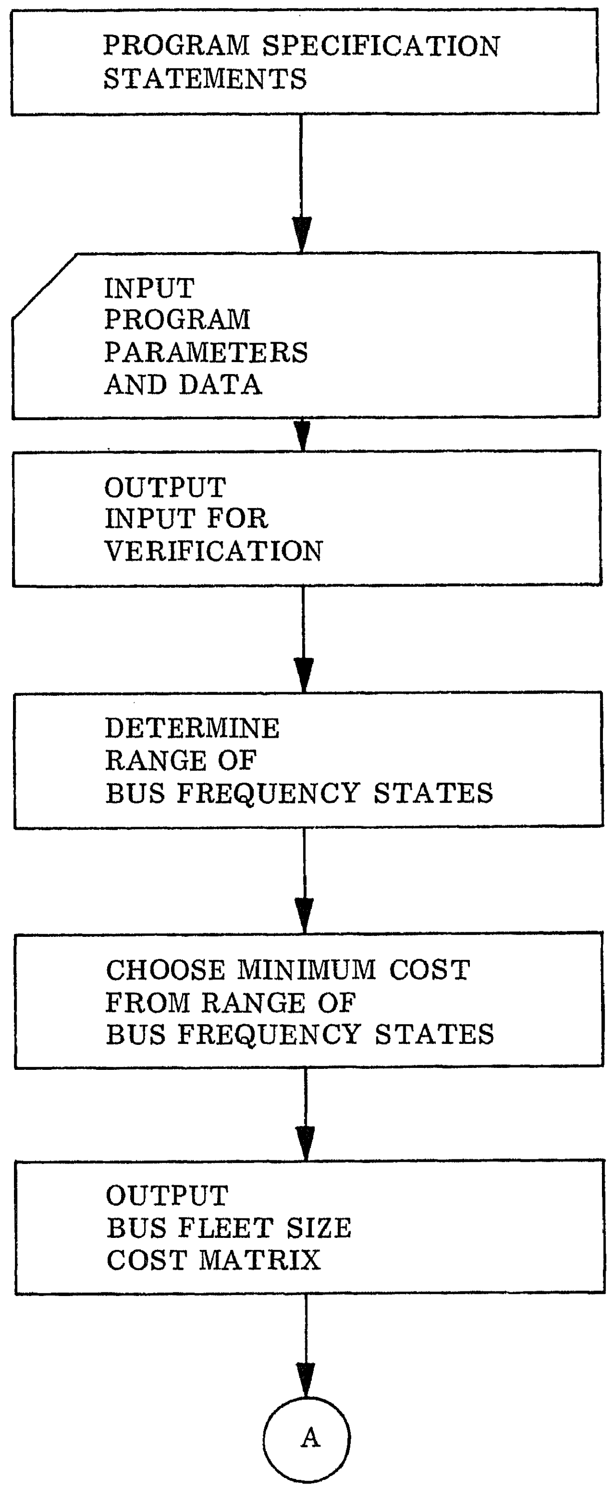

B
SAME AS (SERELA)
EXCEPT LEVEL OF SERVICE
(2) IS DUPLICATE OF
LEVEL OF SERVICE (1)
MULTIPLY (A) BY LINK
TRAVEL TIME ROW
VECTOR, T
TRANSFORM RESULTANT
ROW VECTOR OF TIMES
INTO ROW VECTOR OF COST
BY MULTIPLYING BY COST
FACTOR, CFACT (PCOST)
MULTIPLY (A) BY
LINK OPERATING COST
ROW VECTOR, C
MULTIPLY APPROPRIATE
SECTION OF RESULTANT
ROW VECTOR BY L_1^{-1} OR L_2^{-1}
TO GENERATE (BUSCO)
C

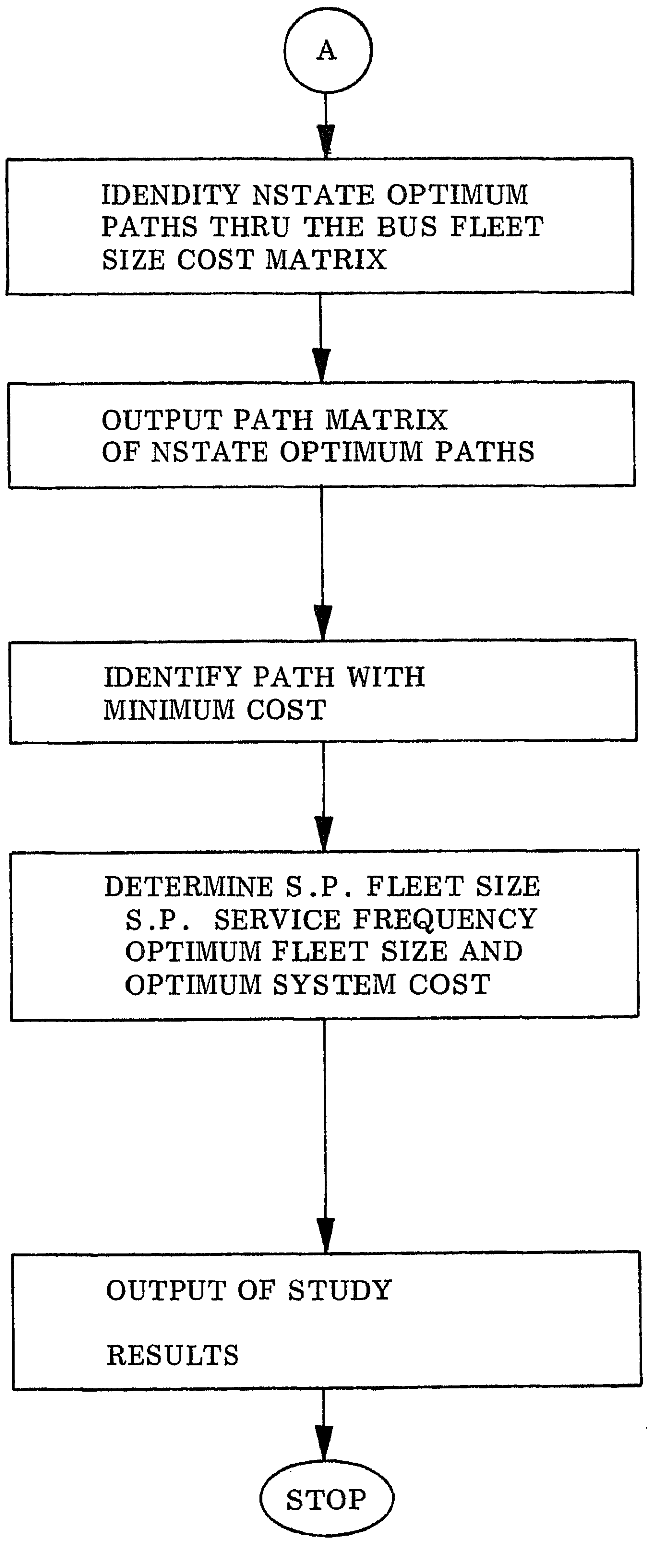

C
PUT COST OF FARE INTO
EACH ELEMENT OF ROW
VECTOR (REV)
GENERATE COST COEFFICIENTS
OBJFUN = (PCOST)
+ (BUSCO) − (REV)
OUTPUT DATA
DATA FILE IS
(LEE . LPDGEN . DATA)
STOP

BUS TRANSIT PLANNING MODEL

FLOW-CHART FOR DYNAMIC PROGRAMMING MODEL (LEEDP)

June, 1973




A
IDENDITY NSTATE OPTIMUM PATHS THRU THE BUS FLEET SIZE COST MATRIX
OUTPUT PATH MATRIX OF NSTATE OPTIMUM PATHS
IDENTIFY PATH WITH MINIMUM COST
DETERMINE S.P. FLEET SIZE S.P. SERVICE FREQUENCY OPTIMUM FLEET SIZE AND OPTIMUM SYSTEM COST
OUTPUT OF STUDY RESULTS
STOP

```
C
C
      PROGRAM LPGEN
C
C
      REAL LNKSER(20,76),LODFT1,LODFT2
C
C
      LOGICAL VERIFY/.FALSE./
C
C
      DIMENSION SERELA(20,76),A(20,38)
      DIMENSION NBRNCH(38),LBRNCH(10,38)
      DIMENSION FLEET(76),BUDGET(38),T(38),C(38)
      DIMENSION PASSD(20,76),PCOST(76),BUSCD(76),NCHAIN(38),REV(76)
      DIMENSION OBJFUN(76)
C
C
      DATA SERELA/1520*0.0/,A/760*0.0/,Y/'Y'/
C
C
C  INPUT DATA FOR LP DATA GENERATOR PROGRAM
C
C
      WRITE(2,10)
   10 FORMAT(' DO YOU WISH TO VERIFY INPUT DATA (Y,N)?')
      READ(1,11),YES
   11 FORMAT(A1)
      IF(YES.EQ.Y) VERIFY=.TRUE.
      READ(75,27) IZ,M,N,LE,IE,IG
   27 FORMAT(6I3)
      READ(75,1) NDMAND
      READ(75,1) NDCHAN
      READ(75,1) NLINKS
    1 FORMAT(I5)
      READ(75,2) LODFT1
      READ(75,2) LODFT2
    2 FORMAT(F8.1)
      ALPHA1=1./LODFT1
      ALPHA2=1./LODFT2
      READ(75,3) (NCHAIN(I),I=1,NDMAND)
    3 FORMAT(10I3)
      K=NDCHAN/2
      DO 4 J=1,K
      READ(75,6) NBRNCH(J)
      L=NBRNCH(J)
    4 READ(75,26) (LBRNCH(I,J),I=1,L)
    6 FORMAT(I3)
   26 FORMAT(10I3)
      READ(75,7) PASSCT
```

```
    7 FORMAT(F5.2)
      CFACT=PASSCT/60.
      DO 8 L=1,NLINKS
    8 READ(75,9) T(L),C(L)
    9 FORMAT(2F7.3)
      READ(75,7) FARE
      WRITE(77,28) IZ,M,N,LE,IE,IG
   28 FORMAT(1X,6I4)
      WRITE(77,29) NDMAND,NLINKS,NDCHAN
   29 FORMAT(1X,3I3)
      IF(.NOT.VERIFY) GO TO 99
      WRITE(2,12) NDMAND,NDCHAN
   12 FORMAT(/' DATA VERIFICATION'
     %//' THERE ARE ',I3,' DEMANDS'
     %/' AND ',I3,' DEMAND-CHAINS'
     %//' DEMAND# CHAIN#     LINKS')
      ND=0
      DO 15 M=1,NDMAND
      MM=NCHAIN(M)
      DO 15 N=1,MM
      ND=ND+1
      L=NBRNCH(ND)
   15 WRITE(2,16) M,N,(LBRNCH(I,ND),I=1,L)
   16 FORMAT(1X,I4,4X,I4,3X,I4,10I4)
      WRITE(2,17) NLINKS
   17 FORMAT(//' THERE ARE ',I3,' LINKS'
     %//1X,'LINK#  TRAVEL TIME  OPERATING COST')
      DO 18 L=1,NLINKS
   18 WRITE(2,19) L,T(L),C(L)
   19 FORMAT(1X,I4,3X,F7.3,6X,F7.3)
      WRITE(2,20) LODFT1,LODFT2,PASSCT,FARE
   20 FORMAT(//' LOAD FACTOR 1= ',F5.1
     %/' LOAD FACTOR 2= ',F5.1
     %/' PASSENGER COST= ',F5.2
     %/' FARE= ',F5.2)
      WRITE(2,21)
   21 FORMAT(///' IS INPUT DATA CORRECT (Y,N)?')
      READ(1,11) YES
      IF(YES.EQ.Y) GO TO 99
      WRITE(2,22)
   22 FORMAT(//' YOU MUST CORRECT YOUR INPUT FILE USING EDT'
     %/' LPDGEN WILL TERMINATE, YOU MUST THEN EXECUTE EDT'
     %/' AND CORRECT YOUR FILE'
     %/' THEN RE-EXECUTE LPDGEN'
     %/' HOWEVER, YOU MUST FIRST ERASE THE PRESENT OBJECT MODUAL'
     %/' BY TYPING ER * FOLLOWING THE SLASH,/, RETURNED BY THE'
     %/' COMPUTER FOLLOWING LPDGEN TERMINATION')
      GO TO 1000
C
C
```

```
C
C   SERVICE ELASTICITY /SERELA/ (NDMAND X NDCHAN)
C
C
C   PUT 1'S FOR EACH CHAIN OF CORRESPONDING ROW DEMAND, 0'S ELSEWHERE
C
C
   99 L=1
      DO 100 I=1,NDMAND
      K=NCHAIN(I)
      DO 100 J=1,K
      SERELA(I,L)=1.0
  100 L=L+1
      WRITE(77,101) ((SERELA(I,J),J=1,NDCHAN),I=1,NDMAND)
  101 FORMAT(1X,8F7.2)
C
C
C   LINK SERVICE CAPACITY /LNKSER/ (NLINKS X NDCHAN) REAL
C
C
C   FIRST GENERATE INCIDENCE MATRIX, /A/, (NLINKS X NDCHAN/2)
C
C
      K=NDCHAN/2
      DO 200 J=1,K
      L=NBRNCH(J)
      DO 200 I=1,L
  200 A(LBRNCH(I,J),J)=1.0
C
C
C   NEXT MULTIPLY /A/ BY ALPHA 1 TO GET THE LEVEL
C   OF SERVICE 1 SECTION OF /LNKSER/ AND /A/ BY ALPHA2
C   TO GET THE LEVEL OF SERVICE 2 SECTION OF /LNKSER/
C
C
      L=NLINKS-3
      DO 210 I=1,L
      DO 210 J=1,K
      LNKSER(I,J)=A(I,J)*ALPHA1
  210 LNKSER(I,J+K)=A(I,J)*ALPHA2
      L=L+1
      DO 211 I=L,NLINKS
      DO 211 J=1,K
      LNKSER(I,J)=A(I,J)
  211 LNKSER(I,J+K)=A(I,J)
      WRITE(77,212) ((LNKSER(I,J),J=1,NDCHAN),I=1,NLINKS)
  212 FORMAT(1X,8F7.2)
C
C
C   FLEET SIZE /FLEET/ (1 X NDCHAN)
```

```
C
C
C   FIRST MULTIPLY INCIDENCE MATRIX, /A/, BY LINK
C   TRAVEL TIME ROW VECTOR, T
C
C
      I=NDCHAN/2
      DO 310 J=1,I
      SUM=0.0
      DO 300 L=1,NLINKS
  300 SUM=SUM+T(L)*A(L,J)
C
C
C   SUBTRACT COST OF IMAGINARY LINKS 12, 13, AND 14
C
C
      DO 305 L=1,3
      K=(NLINKS+1)-L
  305 SUM=SUM-T(K)*A(K,J)
C
C
C   THEN MULTIPLY THE RESULTANT ROW VECTOR BY APPROPRIATE
C   ALPHA 1 OR ALPHA 2
C
C
      FLEET(J)=SUM*ALPHA1
  310 FLEET(J+I)=SUM*ALPHA2
      WRITE(77,311) (FLEET(I),I=1,NDCHAN)
  311 FORMAT(1X,8F7.2)
C
C
C   BUDGET /BUDGET/ (1 X NDCHAN)
C
C
C   FIRST MULTIPLY INCIDENCE MATRIX, /A/, BY LINK
C   OPERATING COST ROW VECTOR, C
C
C
      I=NDCHAN/2
      DO 410 J=1,I
      SUM=0.0
      DO 400 L=1,NLINKS
  400 SUM=SUM+C(L)*A(L,J)
C
C
C   THEN MULTIPLY THE RESULTANT ROW VECTOR BY APPROPRIATE
C   ALPHA 1 OR ALPHA 2
C
C
      BUDGET(J)=SUM*ALPHA1
```

```
  410 BUDGET(J+I)=SUM*ALPHA2
      WRITE(77,411) (BUDGET(I),I=1,NDCHAN)
  411 FORMAT(1X,8F7.2)
C
C
C  PASSENGER DEMAND /PASSD/ (NDMAND X NDCHAN)
C
C
C  SAME AS /SERELA/ EXCEPT LEVEL OF SERVICE 2 SECTION
C  IS DUPLICATE OF LEVEL OF SERVICE 1 SECTION
C  OF /SERELA/
C
C
      K=NDCHAN/2
      DO 500 I=1,NDMAND
      DO 500 J=1,K
      PASSD(I,J)=SERELA(I,J)
  500 PASSD(I,J+K)=SERELA(I,J)
C
      WRITE(77,501) ((PASSD(I,J),J=1,NDCHAN),I=1,NDMAND)
  501 FORMAT(1X,8F7.2)
C
C
C  OBJECTIVE FUNCTION /OBJFUN/ (1 X NDCHAN)
C  OBJECTIVE FUNCTION CONSISTS OF THREE PARTS:
C  MINIMIZE PASSENGER COST PLUS BUS SYSTEM OPERATING COST
C  MINUS REVENUE
C
C
C  FIRST CALCULATE PASSENGER COST /PCOST/ (1 X NDCHAN)
C  MULTIPLY INCIDENCE MATRIX, /A/, BY LINK
C  TRAVEL TIME ROW VECTOR, T
C
C
      K=NDCHAN/2
      DO 610 J=1,K
      SUM=0.0
      DO 600 L=1,NLINKS
  600 SUM=SUM+T(L)*A(L,J)
C
C
C  THEN TRANSFORM THE RESULTING ROW VECTOR OF TIMES
C  INTO A ROW VECTOR OF COSTS BY MULTIPLYING BY
C  COST CONVERSION FACTOR, CFACT  (CFACT=PASSCT/60.)
C
C
      PCOST(J)=SUM*CFACT
  610 PCOST(J+K)=SUM*CFACT
C
C
```

```
C   CALCULATE BUS SYSTEM OPERATING COST /BUSCO/ (1 X NDCHAN)
C   MULTIPLY INCIDENCE MATRIX, /A/, BY LINK OPERATING
C   COST ROW VECTOR, C
C
C
        K=NDCHAN/2
        DO 630 J=1,K
        SUM=0.0
        DO 620 L=1,NLINKS
    620 SUM=SUM+C(L)*A(L,J)
C
C
C   THEN MULTIPLY THE RESULTANT ROW VECTOR BY THE APPROPRIATE
C   ALPHA 1 OR ALPHA 2 TO GET SYSTEM COST
C
C
        BUSCO(J)=SUM*ALPHA1
    630 BUSCO(J+K)=SUM*ALPHA2
C
C
C   CALCULATE REVENUE /REV/ (1 X NDCHAN)
C  PUT COST OF FARE IN EACH ELEMENT OF ROW VECTOR, REV
C
C
        DO 640 I=1,NDCHAN
    640 REV(I)=FARE
C
C
C   COMBINE TERMS TO GET OBJECTIVE FUNCTION
C
C
        DO 650 I=1,NDCHAN
    650 OBJFUN(I)=PCOST(I)+BUSCO(I)-REV(I)
        WRITE(77,651) (OBJFUN(I),I=1,NDCHAN)
    651 FORMAT(1X,8F7.2)
C
C
C   TERMINATE PROGRAM
C
C
   1000 STOP
        END
```

```
      PROGRAM LEEDP
C
C    DYNAMIC PROGRAMMING MODEL
C
      INTEGER DELTA,THETA,PATH(10,10)
C
      DIMENSION DTFACT(10),STFACT(10),BUSFRQ(5,10)
      DIMENSION COST(10,10),SCOST(10),TEXT(10,10)
      DIMENSION NRANGE(10,10),NFLEET(10),NFREQ(10)
C
      WRITE(2,1)
1     FORMAT(20X,'DYNAMIC PROGRAMMING MODEL'//28X,'YOUNG LE
C
C    INPUT PROGRAM PARAMETERS
C
      READ(70,3) M,DELTA,THETA,NSTAGE,NSTATE,OWC
3     FORMAT(5I3,F9.2)
      READ(70,5) (DTFACT(I),I=1,NSTAGE)
5     FORMAT(3F5.2)
      READ(70,5) (STFACT(I),I=1,NSTAGE)
      M=M+1
      READ(70,9) ((BUSFRQ(I,J),I=1,M),J=1,NSTAGE)
9     FORMAT(4F11.2)
      DO 12 I=1,NSTAGE
12    READ(70,10) (TEXT(I,J),J=1,10)
10    FORMAT(10A1)
C
C    VERIFY INPUT DATA
C
      WRITE(2,11) M,DELTA,THETA,NSTAGE,NSTATE,OWC
11    FORMAT(10X,'INPUT DATA'//' M=',I3,14X,'DELTA=',I3
     %/' THETA=',I3,10X,'NSTAGE=',I3/' NSTATE=',I3,9X,'OWC= $',F9.2)
      WRITE(2,8)
8     FORMAT(' STAGE   DELTA/THETA   STAGE FACTOR')
      DO 14 II=1,NSTAGE
      I=(NSTAGE+1)-II
14    WRITE(2,13) I,DTFACT(II),STFACT(II)
13    FORMAT(3X,I1,5X,F6.2,9X,F6.2)
      WRITE(2,15) (I,(BUSFRQ(I,J),J=1,NSTAGE),I=1,M)
15    FORMAT(//20X,'BUS FREQUENCY COSTS'
     %/' STATE/STAGE 6        5         4         3'
     %,'         2         1',5(/3X,I1,4X,6F9.0))
C
C    GENERATE ELEMENT COSTS FOR BUS FLEETSIZE MATRIX
C
C    DETERMINE RANGE OF CHOICES FROM BUS FREQUENCY COSTS
C    EACH ELEMENT HAS
C
      DO 20 J=1,NSTAGE
      DO 20 I=1,NSTATE
      N=(I-1)*THETA*DTFACT(J)/DELTA+1
      IF(N.GT.M) N=M
      JJ=(NSTAGE+1)-J
      WRITE(2,17) I,JJ,N
```

```
17    FORMAT(' FLEETSIZE(',I3,',',I3,') HAS RANGE OF',I3)
C
C  CHOOSE MINIMUM COST FROM RANGE OF COSTS FOR EACH ELEMENT
C
      COST(I,J)=BUSFRQ(1,J)
      NRANGE(I,J)=1
      IF(N.EQ.1) GO TO 20
      DO 25 L=1,N
      IF(COST(I,J).LE.BUSFRQ(L,J)) GO TO 25
      NRANGE(I,J)=L
      COST(I,J)=BUSFRQ(L,J)
25    CONTINUE
20    CONTINUE
C
C  VERIFY BUS FLEETSIZE COST MATRIX
C
      WRITE(2,21) (I,(COST(I,J),J=1,NSTAGE),I=1,NSTATE)
21    FORMAT(//20X,'BUS FLEETSIZE COST MATRIX'
     %/' STATE/STAGE 6         5         4         3'
     %,'         2         1',10(/3X,I1,4X,6F9.0))
      WRITE(2,23)
23    FORMAT(/20X,'CALCULATION ======>'//)
C
C  ADD COST OF INITIAL FLEET SIZE
C
      DO 22 I=1,NSTATE
22    COST(I,NSTAGE)=COST(I,NSTAGE)+(I-1)*THETA*OWC
C
C  IDENTIFY NSTATE OPTIMUM PATHS THRU THE BUS FLEETSIZE MATRIX
C  NOTE:  PROCESS BEGINS AT STAGE N AND MOVES TOWARDS STAGE 1
C  NOTE:  THE NUMBER OF BUSES (THETA'S) CANNOT INCREASE, THEY
C  MUST DECREASE OR REMAIN THE SAME
C
      DO 60 J=2,NSTAGE
      DO 40 I=1,NSTATE
40    SCOST(I)=(I-1)*THETA*OWC*STFACT(J-1)
      DO 60 I=1,NSTATE
      TEMP=10**8
      SAVE=COST(I,J)
      DO 50 II=I,NSTATE
      COST(I,J)=COST(II,J-1)+SCOST(II-I+1)+SAVE
      IF(TEMP.LE.COST(I,J)) GO TO 50
      TEMP=COST(I,J)
      K=NSTAGE-(J-1)
      PATH(I,K)=II
50    CONTINUE
      COST(I,J)=TEMP
60    CONTINUE
C
C  OUTPUT PATH MATRIX
C
      K=NSTAGE-1
      WRITE(2,61) (I,(PATH(I,J),J=1,K),I=1,NSTATE)
61    FORMAT(10X,'PATH MATRIX'
```

```
      %/' STATE/STAGE  1  2  3  4  5'
      %10(/3X,I1,8X,5I3))
C
C     FIND PATH WITH MINIMUM COST OF THE NSTATE PATHS
C
      XMIN=COST(1,NSTAGE)
      DO 70 I=2,NSTATE
      IF(XMIN.LE.COST(I,NSTAGE)) GO TO 70
      XMIN=COST(I,NSTAGE)
      IMIN=I
70    CONTINUE
C
C     OUTPUT RESULTS
C
      WRITE(2,73)
73    FORMAT(//1X,11('* '),'BUS SYSTEM STUDY RESULTS',11(' *'))
      NFLEET(1)=(IMIN-1)*THETA
      MAXF=NFLEET(1)
      NFREQ(1)=(NRANGE(IMIN,6)-1)*DELTA
      DO 78 J=2,NSTAGE
      IMIN=PATH(IMIN,J-1)
      NFLEET(J)=(IMIN-1)*THETA
      IF(NFLEET(J).LE.MAXF) GO TO 77
      MAXF=NFLEET(J)
77    K=NSTAGE-(J-1)
      NFREQ(J)=(NRANGE(IMIN,K)-1)*DELTA
78    CONTINUE
      IF(MAXF.GT.0) GO TO 83
      WRITE(2,80)
80    FORMAT(//' PROPOSED BUS ROUTE NOT RECOMMENDED')
      STOP
83    WRITE(2,85) (((TEXT(J,I),I=1,10),NFLEET(J)),J=1,NSTAGE)
85    FORMAT(//10X,'SCHEDULE PERIOD FLEET SIZE'
     %6(/1X,10A1,5X,I4,' BUSES'))
      WRITE(2,87) (((TEXT(J,I),I=1,10),NFREQ(J)),J=1,NSTAGE)
87    FORMAT(//1X,'SCHEDULE PERIOD SERVICE FREQUENCY'
     %6(/1X,10A1,5X,I4,' DISPATCHES'))
      WRITE(2,89) MAXF,XMIN
89    FORMAT(//' OPTIMUM FLEET SIZE OF PROPOSED BUS ROUTE IS',I5,
     %' BUSES'//' OPTIMUM TOTAL BUS TRANSIT SYSTEM COST IS $',F15.2///)
      STOP
      END
```

```
      PROGRAM LPTEST
      DIMENSION ROW(50),SROW(50),COL(75),A(50,75),RHS(50)
      DATA UMINUS/'-'/,BLANK/' '/
      WRITE(81,10)
10    FORMAT(50('*'))
      WRITE(81,1)
1     FORMAT('12345678901234567890123456789012345678901234567890')
      READ(80,2) (SROW(I),ROW(I),I=1,39)
2     FORMAT(A1,A4)
      READ(80,12) (COL(I),I=1,66)
12    FORMAT(A4)
      WRITE(81,3) (SROW(I),ROW(I),I=1,39)
3     FORMAT(1X,A1,2X,A4)
5     READ(79,9) ((A(I,J),J=1,66),I=2,12)
6     READ(79,9) ((A(I,J),J=1,66),I=13,26)
7     READ(79,9) (A(27,J),J=1,66)
      READ(79,9) (A(28,J),J=1,66)
8     READ(79,9) ((A(I,J),J=1,66),I=29,39)
      READ(79,9) (A(1,J),J=1,66)
9     FORMAT(1X,8F7.2)
      RHS(1)=0.0
      READ(79,11) (RHS(I),I=2,39)
11    FORMAT(1X,5F10.2)
      WRITE(81,10)
      DO 20 J=1,66
      DO 20 I=1,39
      IF(A(I,J).EQ.0.00) GO TO 20
18    WRITE(81,25) COL(J),ROW(I),A(I,J)
25    FORMAT(4X,A4,6X,A4,6X,F12.3)
20    CONTINUE
      WRITE(81,10)
      DO 26 I=2,39
21    WRITE(81,27) ROW(I),RHS(I)
27    FORMAT(4X,'RHS01',5X,A4,6X,F12.3)
26    CONTINUE
      WRITE(81,10)
      STOP
      END
```

```
/PROC C
/FILE LEE.LPOGEN.DATA,LINK=DSET79,FCBTYPE=SAM,RECFORM=V
/FILE LEE.LPTEST.INPUT,LINK=DSET80,FCBTYPE=SAM,RECFORM=V
/FILE LEE.LPTEST.OUTPUT,LINK=DSET81,FCBTYPE=SAM,RECFORM=V
/EXEC LPTEST
/PRINT LEE.LPTEST.
/ENDP
```

APPENDIX D

SUPPLEMENTAL DATA

NO	DEMAND	EXPECTED PASSENGER FORECAST(D_i)						LIMIT OF LOAD FACTOR 1 (U_i)					
	O-D \ S.P.	1	2	3	4	5	6	1	2	3	4	5	6
1	1-3	572	432	603	226	361	373	458	346	482	181	289	298
2	1-5	1472	1111	1552	586	929	961	1178	889	1242	469	743	769
3	1-6	793	597	835	214	500	516	634	478	668	172	400	413
4	1-7	1418	1080	1492	564	896	925	1135	864	1193	451	727	740
5	3-7	413	312	435	164	260	269	330	250	348	131	208	215
6	3-8	674	713	995	376	596	616	755	570	796	300	477	493
7	5-7	780	588	822	310	492	508	624	471	658	248	394	407
8	5-8	1650	1246	1741	654	1043	1077	1320	997	1393	523	834	861
9	6-8	1174	887	1237	464	742	766	939	709	989	371	594	613
10	7-8	1561	1179	1646	618	987	1019	1249	943	1317	494	790	815
11	7-20	530	400	558	210	335	345	424	320	446	168	268	276

TABLE 8. PASSENGER DEMAND AND LIMIT OF LOAD FACTOR 1

(3) WEEK DAY (12 hours)
OFF PEAK

O \ D	1	3	5	6	7	8	15	17	20
1		603	1552	835	1492				
3					435	995			
5					822	1741			
6						1237			
7						1646			558
8									
15									
17									
20									

(4) SATURDAY (4 hours)
PEAK PERIOD

O \ D	1	3	5	6	7	8	15	17	20
1		226	586	214	564				
3					164	376			
5					310	654			
6						464			
7						618			210
8									
15									
17									
20									

TABLE 9 BUS PASSENGER DEMAND FORECAST
DURING SCHEDULE PERIODS 3 & 4

(5) WEEK DAY (3 hours)
A.M. PEAK

O \ D	1	3	5	6	7	8	15	17	20
1		361	929	500	896				
3					260	596			
5					492	1043			
6						742			
7						987			335
8									
15									
17									
20									

(6) WEEK DAY (3 hours)
P.M. PEAK

O \ D	1	3	5	6	7	8	15	17	20
1		373	961	516	925				
3					269	616			
5					508	1077			
6						766			
7						1019			345
8									
15									
17									
20									

TABLE 10 BUS PASSENGER DEMAND FORECAST
DURING SCHEDULE PERIODS 5 & 6

WEEKDAYS

TO NEWARK																		TO MORRISTOWN
CHATHAM CENTER Main St. & Passaic Ave.	SHORT HILLS MALL	SUMMIT River Rd. & Iris Rd.	SUMMIT Broad & Elm Sts.	SPRINGFIELD Morris & Mountain Aves.	MILLBURN Millburn Ave. & Lackawanna Pl.	MAPLEWOOD LOOP	HILTON GARAGE	IRVINGTON CENTER	NEWARK Penn Railroad Station	NEWARK Penn Railroad Station	IRVINGTON CENTER	HILTON GARAGE	MAPLEWOOD LOOP	MILLBURN Main St. & Millburn Ave.	SPRINGFIELD Morris & Mountain Aves.	SUMMIT Broad & Elm Sts.	SUMMIT River Rd. & Iris Rd.	SHORT HILLS MALL
-	-	4.55	5.00	5.10	5.13	5.17	5.22	5.27	5.43	-	-	4.30	4.37	4.41	-	-	4.50	-
-	-	-	-	-	-	-	5.45	5.50	6.06	-	-	4.50	4.57	5.01	5.04	5.14	[illegible]	-
-	-	-	-	-	-	-	5.55	6.00	6.16	-	-	5.23	5.30	5.34	-	-	5.43	-
-	-	-	-	-	-	-	6.05	6.11	6.31	-	-	5.25	5.32	5.36	5.39	5.49	5.54	-
-	-	-	-	-	-	-	6.15	6.21	6.41	-	-	5.30	5.37	5.41	-	-	5.50	-
-	-	5.55	6.00	6.10	6.13	6.17	6.24	6.30	6.50	-	-	5.40	5.47	5.53	5.56	6.06	6.13	-
-	-	-	-	-	-	-	6.41	6.47	7.07	-	-	5.57	6.04	6.10	-	-	-	-
6.08	-	6.12	6.18	6.28	6.31	6.35	6.42	6.48	7.08	W5.50	6.03	6.09	6.16	6.22	6.25	6.35	6.42	-
-	-	-	-	-	-	-	7.05	7.11	7.31	5.47	6.10	6.16	6.23	6.29	6.32	6.42	6.49	-
6.30	-	6.34	6.42	6.52	6.56	7.00	7.07	7.13	7.33	-	-	6.20	6.27	6.33	-	-	6.44	-
-	-	6.51	6.59	7.09	7.13	7.17	7.24	7.30	7.50	W6.10	6.23	6.29	6.36	6.42	6.45	6.55	7.02	-
7.07	-	7.11	7.19	7.29	7.34	7.39	7.46	7.52	8.14	-	-	6.34	6.41	6.47	6.50	7.00	7.07	-
-	7.28	7.31	7.39	7.49	7.54	7.59	8.06	8.12	8.34	6.10	6.33	6.39	6.46	6.52	6.56	7.06	7.14	-
7.45	-	7.49	7.57	8.07	8.12	8.17	8.24	8.30	8.52	6.20	6.43	6.49	6.56	7.02	7.06	7.16	7.24	7.26
8.05	-	8.09	8.17	8.27	8.32	8.37	8.44	8.50	9.12	6.34	6.57	7.03	7.10	7.16	7.20	7.30	7.38	-
-	-	-	-	-	-	-	-	-	-	6.46	7.11	7.17	7.24	7.30	7.34	7.44	7.52	-
8.20	8.26	8.29	8.37	8.47	8.52	8.57	9.04	-	-	7.00	7.25	7.31	7.38	7.44	7.48	7.58	8.06	-
8.25	-	8.29	8.37	8.47	8.52	8.57	9.04	9.10	9.32	7.12	7.37	7.43	7.50	7.56	8.00	8.10	8.18	8.20
-	8.46	8.49	-	-	9.00	9.04	9.11	-	-	7.24	7.49	7.55	8.02	8.08	8.12	8.22	8.30	-
-	8.46	8.49	8.57	9.07	9.11	9.15	9.22	9.28	9.48	7.36	8.01	8.07	8.14	8.20	8.24	8.34	8.42	8.44
9.05	-	9.09	9.17	9.27	9.31	9.35	9.42	9.48	10.08	7.48	8.13	8.19	8.26	8.32	8.36	8.46	9.54	-
-	9.16	9.19	9.27	9.37	9.41	9.45	9.52	-	-	8.00	8.25	8.31	8.38	8.44	8.48	8.58	9.06	9.08
-	9.22	9.25	-	-	9.35	9.40	9.47	-	-	8.15	8.38	8.44	8.51	8.57	9.01	9.11	9.19	9.21
9.25	-	9.29	9.37	9.47	9.51	9.55	10.02	10.08	-	8.30	8.53	8.59	9.06	9.12	9.16	9.26	9.34	-
9.40	-	9.44	9.52	10.02	10.06	10.10	10.17	10.23	10.43	8.45	9.08	9.14	9.21	9.27	9.31	9.41	9.49	9.51
-	9.53	9.56	-	-	10.07	10.11	10.18	-	-	9.00	9.23	9.29	9.36	9.42	9.46	9.56	10.04	10.06
9.55	10.01	10.04	10.12	10.22	10.26	10.30	10.37	-	-	9.15	9.38	9.44	-	-	-	-	-	-
10.10	-	10.14	10.22	10.32	10.36	10.40	10.47	10.53	11.13	9.30	9.53	9.59	10.06	10.12	10.16	10.26	10.34	-
-	10.41	10.44	10.52	11.02	11.06	11.10	11.17	11.23	11.43	9.50	10.13	10.19	-	-	-	-	-	-
11.10	-	11.14	11.22	11.32	11.36	11.40	11.47	11.53	12.13	10.00	10.23	10.29	10.36	10.42	10.46	10.56	11.04	11.06
-	11.41	11.44	11.52	12.02	12.06	12.10	12.17	12.23	12.43	10.30	10.53	10.59	11.06	11.12	11.16	11.26	11.34	-
12.10	-	12.14	12.22	12.32	12.36	17.40	12.47	12.53	1.13	11.00	11.23	11.29	11.36	11.42	11.46	11.56	12.04	12.06
-	12.41	12.44	12.52	1.02	1.06	1.10	1.17	1.23	1.43	11.30	11.53	11.59	12.06	12.12	12.16	12.26	12.34	-
1.10	-	1.14	1.22	1.32	1.36	1.40	1.47	1.53	2.13	12.00	12.23	12.29	12.36	12.42	12.46	12.56	1.04	1.06
-	-	-	-	-	-	-	2.07	2.13	2.33	12.30	12.53	12.59	1.06	1.12	1.16	1.26	1.34	-

SATURDAYS

TO NEWARK																		TO MORRISTOWN
CHATHAM CENTER Main St. & Passaic Ave.	SHORT HILLS MALL	SUMMIT River Rd. & Iris Rd.	SUMMIT Broad & Elm Sts.	SPRINGFIELD Morris & Mountain Aves.	MILLBURN Millburn Ave. & Lackawanna Pl.	MAPLEWOOD LOOP	HILTON GARAGE	IRVINGTON CENTER	NEWARK Penn Railroad Station	NEWARK Penn Railroad Station	IRVINGTON CENTER	HILTON GARAGE	MAPLEWOOD LOOP	MILLBURN Main St. & Millburn Ave.	SPRINGFIELD Morris & Mountain Aves.	SUMMIT Broad & Elm Sts.	SUMMIT River Rd. & Iris Rd.	SHORT HILLS MALL
-	-	-	-	-	-	-	[illegible]	5.27	[illegible]	-	-	[illegible]	[illegible]	[illegible]	-	-	5.53	-
-	-	-	-	-	-	-	6.02	6.12	[illegible]	-	-	5.20	[illegible]	[illegible]	5.47	[illegible]	6.04	-
-	-	5.57	6.02	6.11	6.16	6.20	6.27	6.32	6.53	-	-	[illegible]	6.27	6.32	-	-	-	-
6.31	-	6.34	6.42	6.52	6.56	7.00	7.07	7.12	[illegible]	5.50	6.13	6.18	6.26	6.32	6.36	6.46	6.54	-
-	-	-	-	-	-	-	7.37	7.42	8.03	6.35	6.58	7.03	7.11	7.17	7.21	7.31	7.39	7.41
7.09	-	7.14	7.22	7.32	7.36	7.40	7.47	7.52	8.13	7.10	7.33	7.38	7.46	7.52	7.56	8.06	8.14	-
-	7.51	7.54	8.02	8.12	8.16	8.20	8.27	8.32	8.53	7.40	8.03	8.08	8.16	8.22	8.26	8.36	8.44	8.46
-	-	-	-	-	-	-	9.04	9.09	9.30	8.10	8.33	8.38	8.46	8.52	8.56	9.06	9.14	9.16
8.29	-	8.34	8.42	8.52	8.56	9.00	9.07	9.12	9.33	8.40	9.03	9.08	9.16	9.22	9.26	9.36	9.44	9.46
-	9.01	9.04	9.12	9.22	9.26	9.30	9.37	9.42	10.03	9.10	9.33	9.38	9.46	9.52	9.56	10.06	10.14	10.16
-	9.31	9.34	9.42	9.52	9.56	10.00	10.07	10.12	10.33	9.35	9.58	10.03	10.11	10.17	10.21	10.31	10.39	10.41
9.55	10.01	10.04	10.12	10.22	10.26	10.30	10.37	10.42	11.03	10.00	10.23	10.28	10.36	10.42	10.46	10.56	11.04	11.06
-	10.27	[illegible]	10.37	10.47	10.51	10.55	11.02	11.07	11.28	10.25	10.48	10.53	11.01	11.07	11.11	11.21	11.29	11.31
-	10.51	[illegible]	[illegible]	11.12	11.16	11.20	11.27	11.32	[illegible]	10.50	11.13	11.18	11.26	11.32	11.36	11.46	11.54	11.56
-	11.16	11.19	11.27	11.37	11.41	11.45	11.52	11.57	12.18	11.15	11.38	11.43	11.51	11.57	12.01	12.11	12.19	12.21
11.35	11.41	11.44	11.52	12.02	12.06	12.10	12.17	12.22	12.43	11.40	12.03	12.08	12.16	12.22	12.26	12.36	12.44	12.46
-	12.06	12.09	12.17	12.27	12.31	12.35	12.42	12.47	1.08	12.05	12.28	12.33	12.41	12.47	12.51	1.01	1.09	1.11
-	12.31	12.34	12.42	12.52	12.56	1.00	1.07	1.12	1.33	12.30	12.53	12.58	1.06	1.12	1.16	1.26	1.34	1.36
-	12.56	12.59	1.07	1.17	1.21	1.25	1.32	1.37	1.59	12.55	1.18	1.23	1.31	1.37	1.41	1.51	1.59	2.01
1.15	1.21	1.24	1.32	1.42	1.46	1.50	1.57	2.02	2.23	1.20	1.43	1.48	1.56	2.02	2.06	2.16	2.24	2.26
-	1.46	1.49	1.57	2.07	2.11	2.15	2.22	2.27	2.48	1.45	2.08	2.13	2.21	2.27	2.31	2.41	2.49	2.51

*Source: Transport of New Jersey July,1972

FIGURE 10 TYPICAL PUBLIC BUS SCHEDULE

DM. / DM-CH / LINK NO		1		2			3 .		.	.	
		11	12	21	22	23	31	32	..	51	52
1	street link	0	0	0	0	0	0	0	..	0	0
2	"	0	0	0	0	0	1	0	..	0	0
3	"	1	0	1	0	0	1	0	..	1	0
4	"	0	0	1	0	0	1	0	..	1	1
5	"	0	0	0	0	0	0	0	..	1	1
6	"	0	0	0	0	0	0	0	..	0	0
7	"	0	0	0	0	0	0	1	..	0	0
8	"	0	0	0	1	0	0	0	..	0	1
9	"	0	0	0	0	0	0	0	..	0	1
10	"	0	0	0	1	0	0	1	..	0	0
11	"	0	0	0	0	0	0	0	..	0	0
12	walk	0	0	0	0	0	0	0	..	0	1
13	trans-fer	0	0	0	0	0	0	0	..	0	0
14	other mode	0	1	0	0	1	0	0	..	0	0

TABLE 11 TYPICAL DEMAND-CHAIN INCIDENCE MATRIX

TRAVEL TIME AND DELAY STUDY

STREET Broad DIRECTION N - Bound TIME START 3-37-30

DAY Monday DATE Oct. 2, 1967 WEATHER Clear TRIP NO 1

INTERSECTION CHECK POINT	TIME CROSSING	REASON FOR DELAY	DELAY TIME	TRAVEL TIME	TOTAL TIME
Miller	3.37.30				
Wright	38.20	T.S	.26	0: 00: 24	0: 00: 50
So. St.	40.15	T.S	.36	0: 01: 19	0: 01: 55
Chest.	37.20	T.S	.28		
Franklin		T.S.	.38		
Laf.	44.10			0: 02: 49	0.03.55
13 Plac.		T.S.	.43		
Market	45.15			0: 00: 22	0: 01: 05
Academy		T.S.	.41		
Raymond	46.15			0: 00: 19	0: 01: 00
Rectur		T.S	.42		
Central	48.30			0: 01: 33	0: 02: 15
Bridge		T.S.	.27		
Orange	50.00				
R. R.	50.30			0: 01: 33	0: 02: 00
Total	13: 00		4: 41	8: 19	13: 00

REASON FOR DELAY:
T.S. = TRAFFIC SIGNAL
CONG. = CONGESTION

DRIVER Siemer OBSERVER Grant

FIGURE 11 TYPICAL SPEED AND DELAY SURVEY FORM

LINK NUMBER	LINK TRAVEL TIME	LINK OPER. COST	CAPACITY CONSTRNT
1	2.270	0.682	200
2	3.060	0.716	150
3	4.330	1.044	150
4	4.550	1.064	210
5	3.220	0.800	210
6	11.130	2.828	200
7	8.690	2.102	100
8	7.430	1.764	100
9	9.160	1.946	100
10	8.480	1.840	250
11	6.700	1.498	250
12	5.000	0.000	-
13	10.000	0.000	-
14	120.000	0.000	-

TABLE 12 LINK PROPERTIES

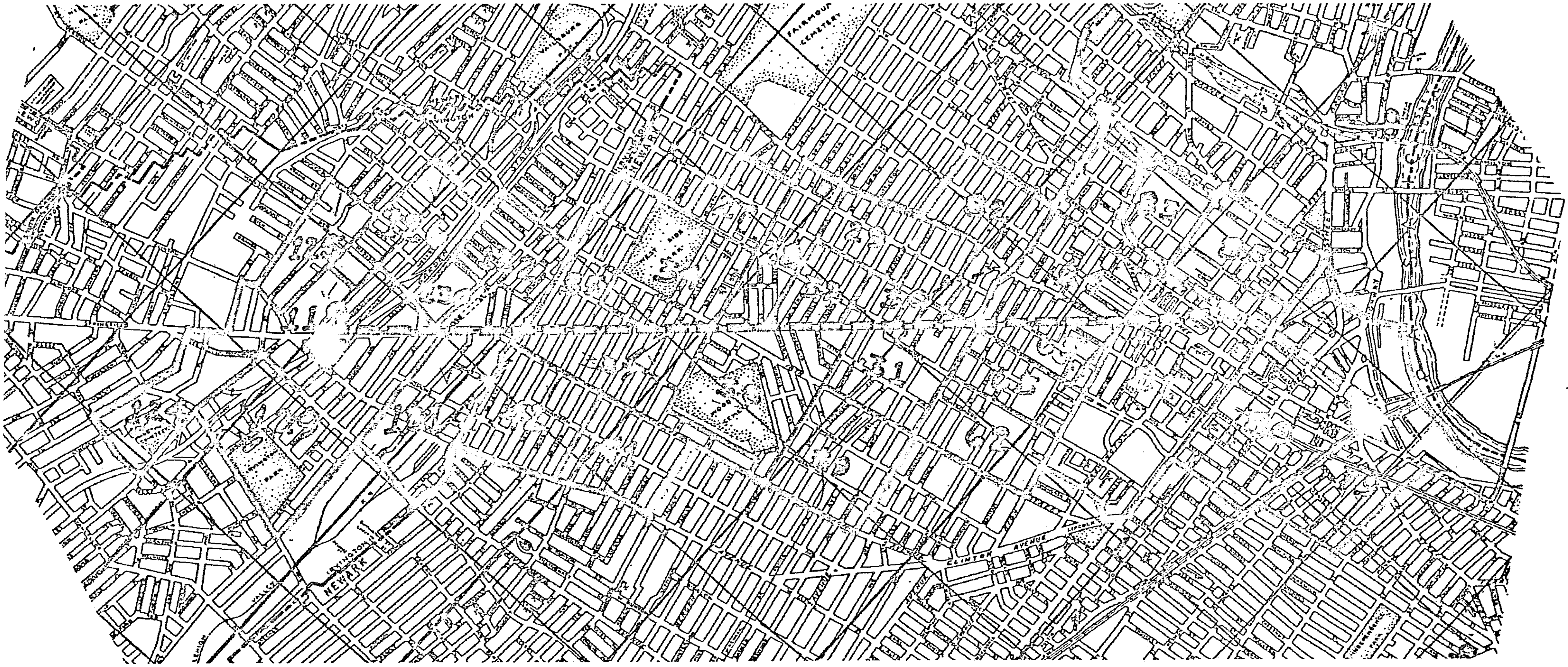
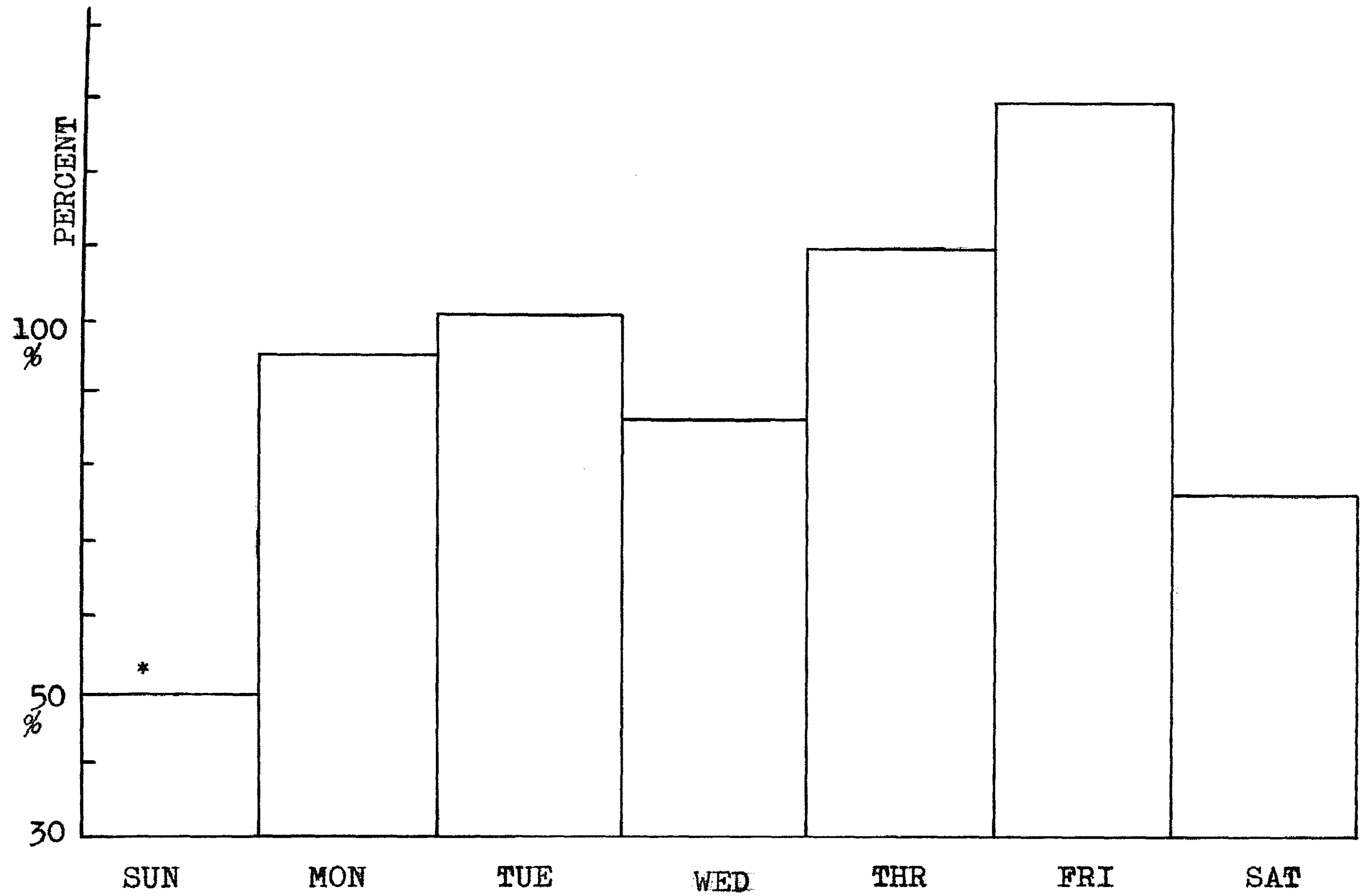


FIGURE 12 TYPICAL CENSUS TRACTS OF SPRINGFIELD AVENUE CORRIDOR IN NEWARK, NEW JERSEY

NO	O-D	D-C	LINK	NO	O-D	D-C	LINK
1	1-3	1,1	2	6	3-8	6,2	14
"	"	1,2	14	7	5-7	7,1	4,5
2	1-5	2,1	2,3	"	"	7,2	14
"	"	2,2	10,8	8	5-8	8,1	13,3,2,1
"	"	2,3	14	"	"	8,2	13,8,10,12,1
3	1-6	3,1	2,3,4	"	"	8,3	14
"	"	3,2	10,7	9	6-8	9,1	4,3,2,1
"	"	3,3	14	"	"	9,2	7,10,1
4	1-7	4,1	2,3,4,5	"	"	9,3	14
"	"	4,2	10,6	10	7-8	10,1	5,4,3,2,1
"	"	4,3	10,7,5	"	"	10,2	5,7,10,1
"	"	4,4	14	"	"	10,3	6,10,12,1
5	3-7	5,1	3,4,5	"	"	10,4	14
"	"	5,2	9,12,8,4,5	11	7-20	11,1	13,5,4,3,2,12,11
"	"	5,3	14	"	"	11,2	13,4,8,10,11
				"	"	11,3	13,6,10,11
6	3-8	6,1	2,1	"	"	11,4	14

TABLE 13 CHAINS OF LINKS



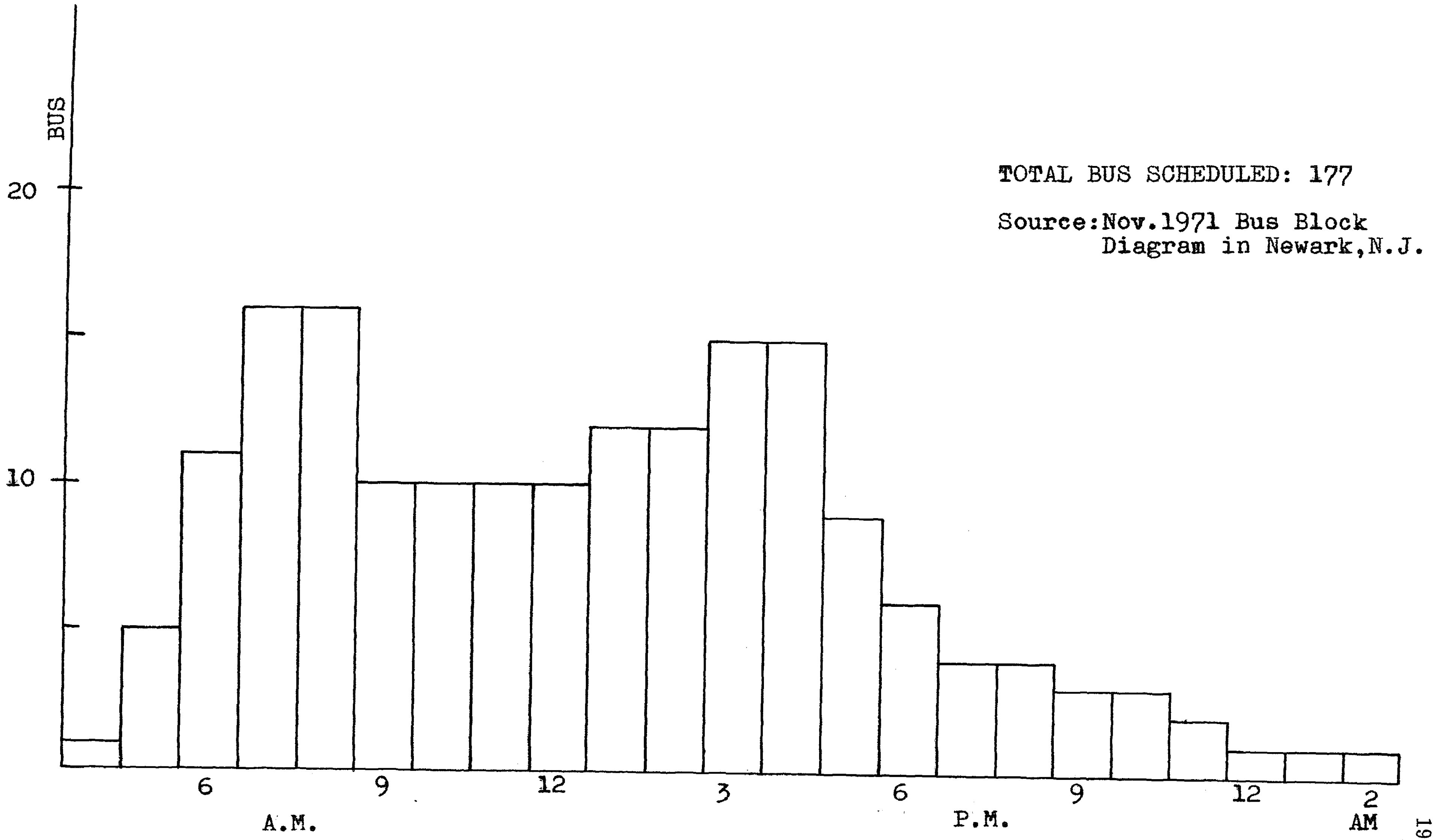
WILLIAMSPORT 1969 PASSENGER DAILY VARIATION
(Based on Fare Collection Statistics)

* Based on Sunday Block Diagram of line # 25-26 in Newark.
Same information is not available for Williamsport.

FIGURE 13

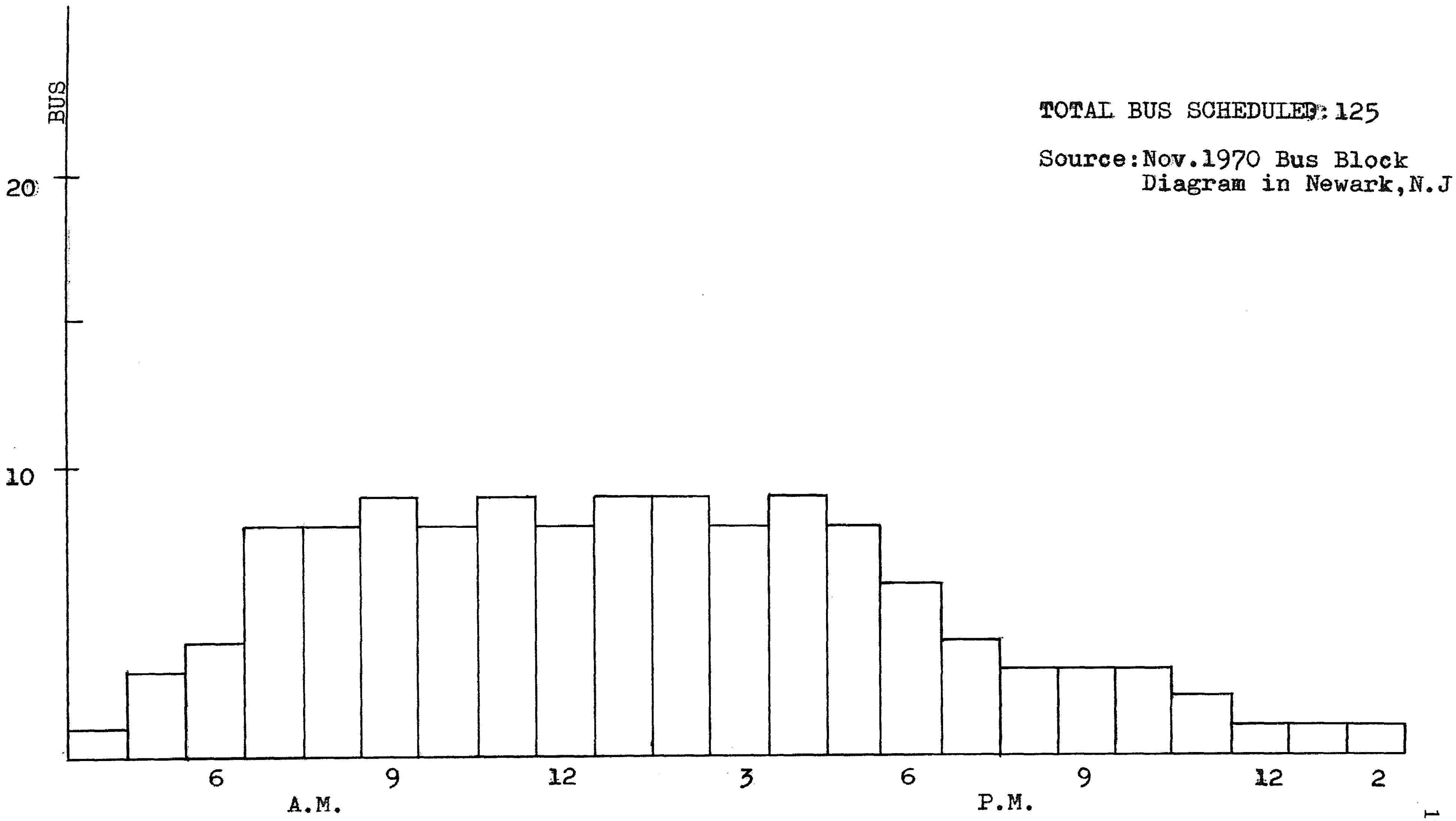
BUS ROUTE		SUN	SAT. OFF	WEEK OFF	SAT PEAK	WEEK A.M.	WEEK P.M.
RT. 13	DISPATCH	104	110	108	40	51	55
	LINK	6,10,11					
RT. 16	DISPATCH	64	70	77	26	35	35
	LINK	6,10,11					
RT. 25	DISPATCH	87	91	93	34	42	41
	LINK	5,4,3,2,1					
RT. 52	DISPATCH	0	0	10	0	10	9
	LINK	5,7,10,11					
RT. 70	DISPATCH	0	0	10	24	26	13
	LINK	5,7,10,1					
ROUTE PROP.							
	LINK	5,4,8,10,11					
FLEET SIZE		N_1	N_2	N_3	N_4	N_5	N_6
SERVICE HOUR		18	18	12	4	3	3
BUDGET ($)		30,000	30,000	20,000	15,000	10,000	10,000

TABLE 14 BUS ROUTE AND SERVICE CONFIGURATIONS



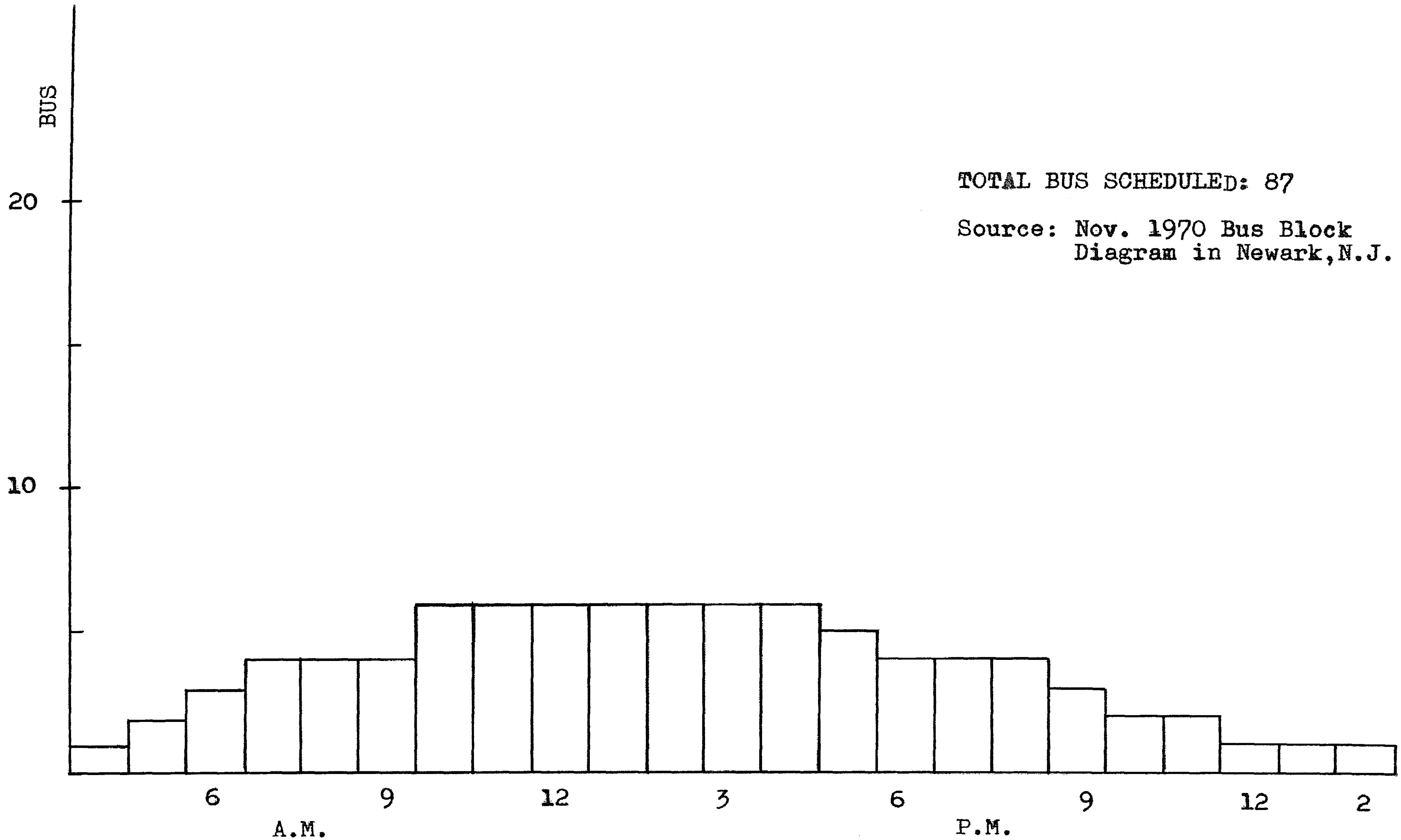
WEEKDAY BUS (# 25-26) SCHEDULE
HOURLY DISTRIBUTION

FIGURE 14



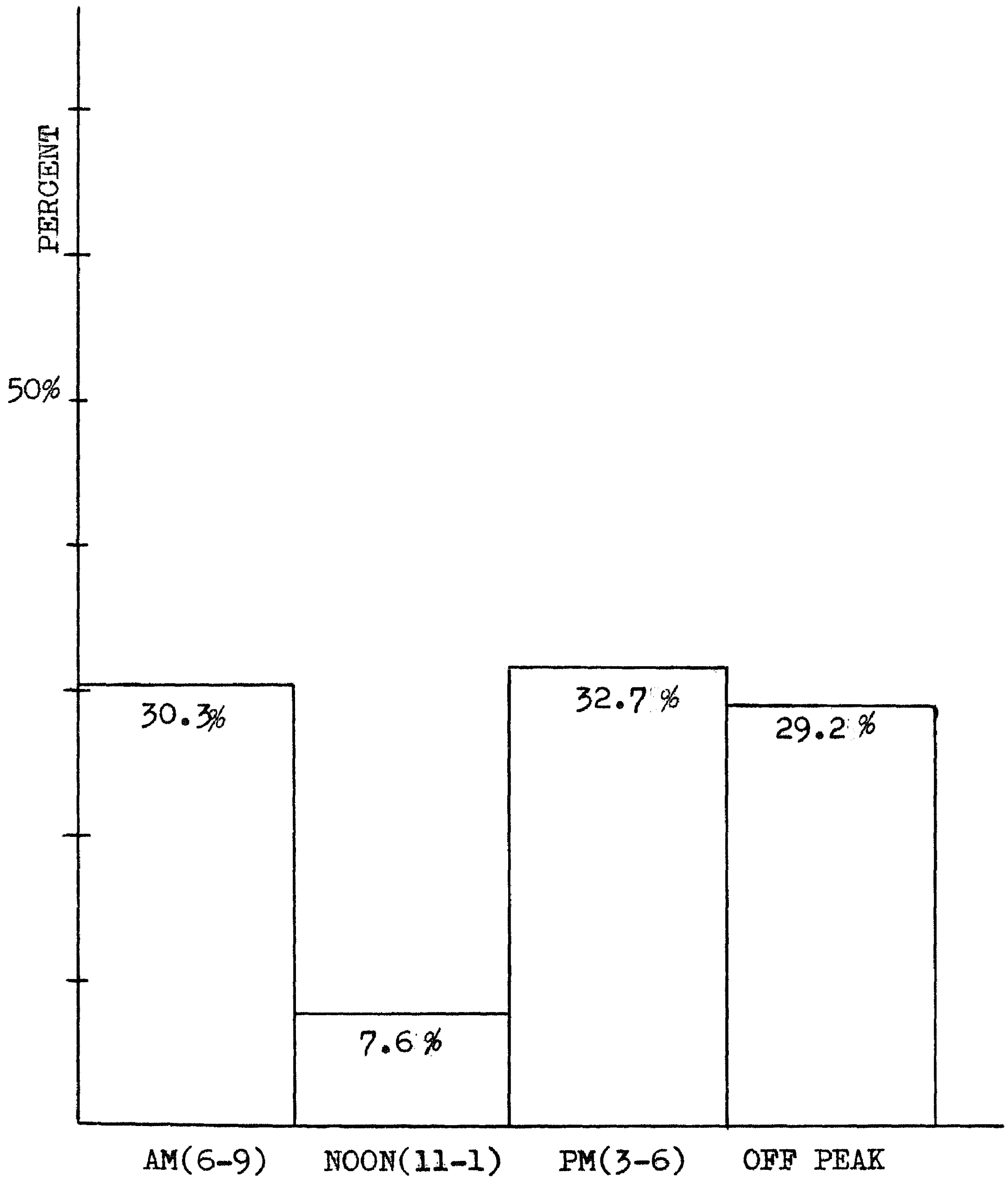
SATURDAY BUS (# 25-26) SCHEDULE
HOURLY DISTRIBUTION

FIGURE 15



SUNDAY BUS (# 25-26) SCHEDULE
HOURLY DISTRIBUTION

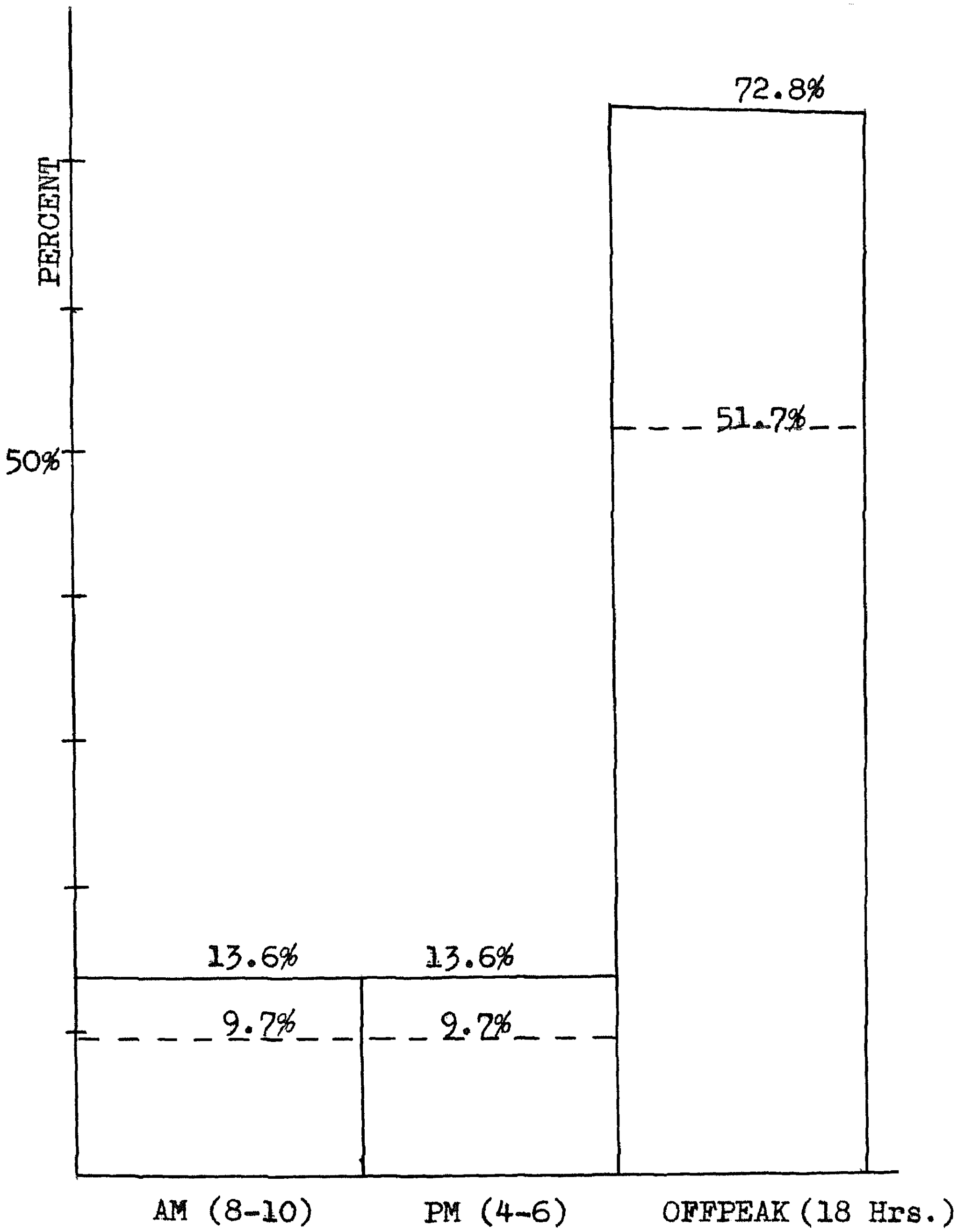
FIGURE 16



WEEKDAY SCHEDULE VARIATION BASED
ON IRVINGTON BUS TERMINAL OPERATION

LEGEND 30.3 .... % of Average Weekday Total

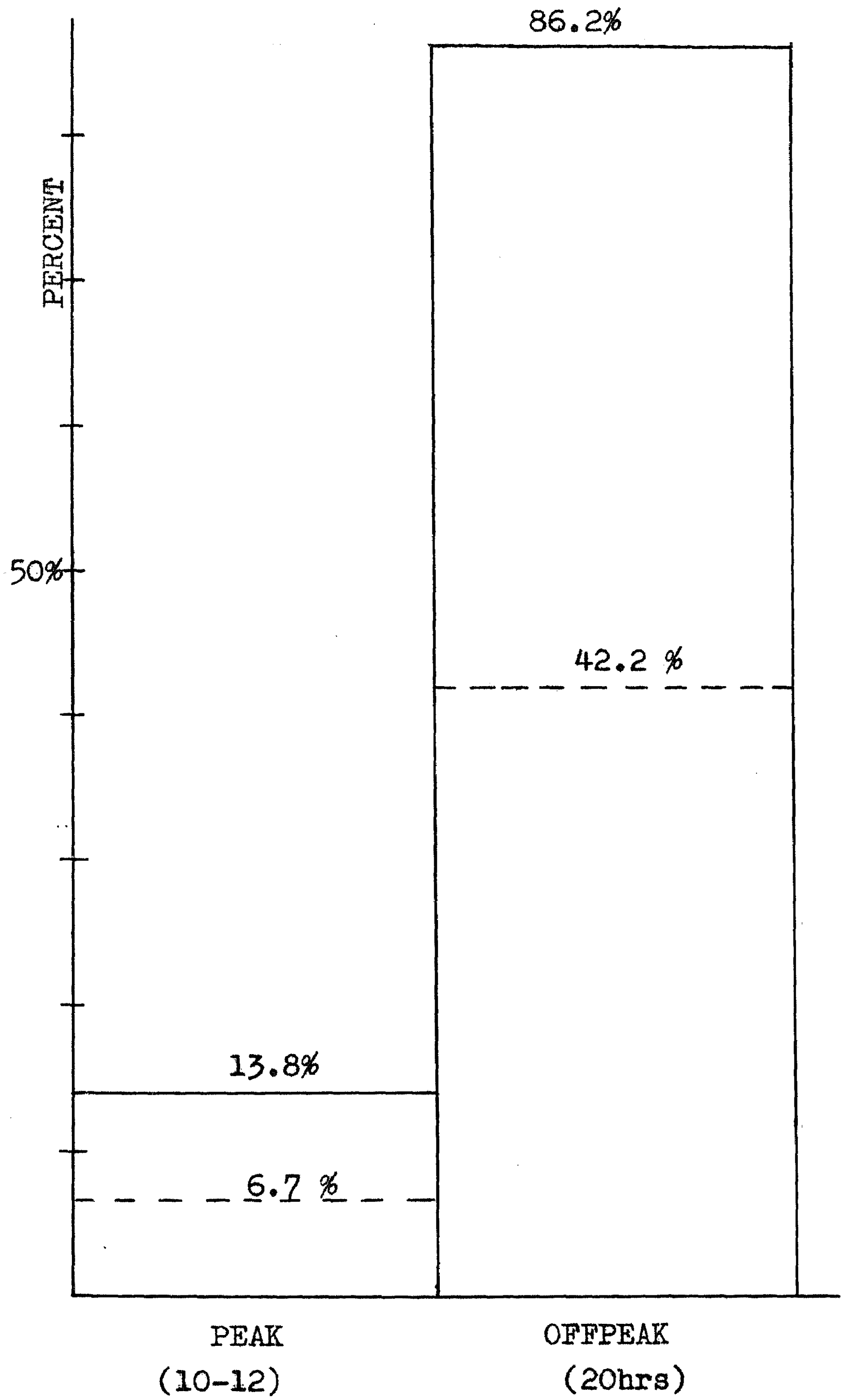
FIGURE 17



SATURDAY SCHEDULE VARIATION

LEGEND 13.6%.... % of Saturday Total
9.7%.... % of Average Weekday Total

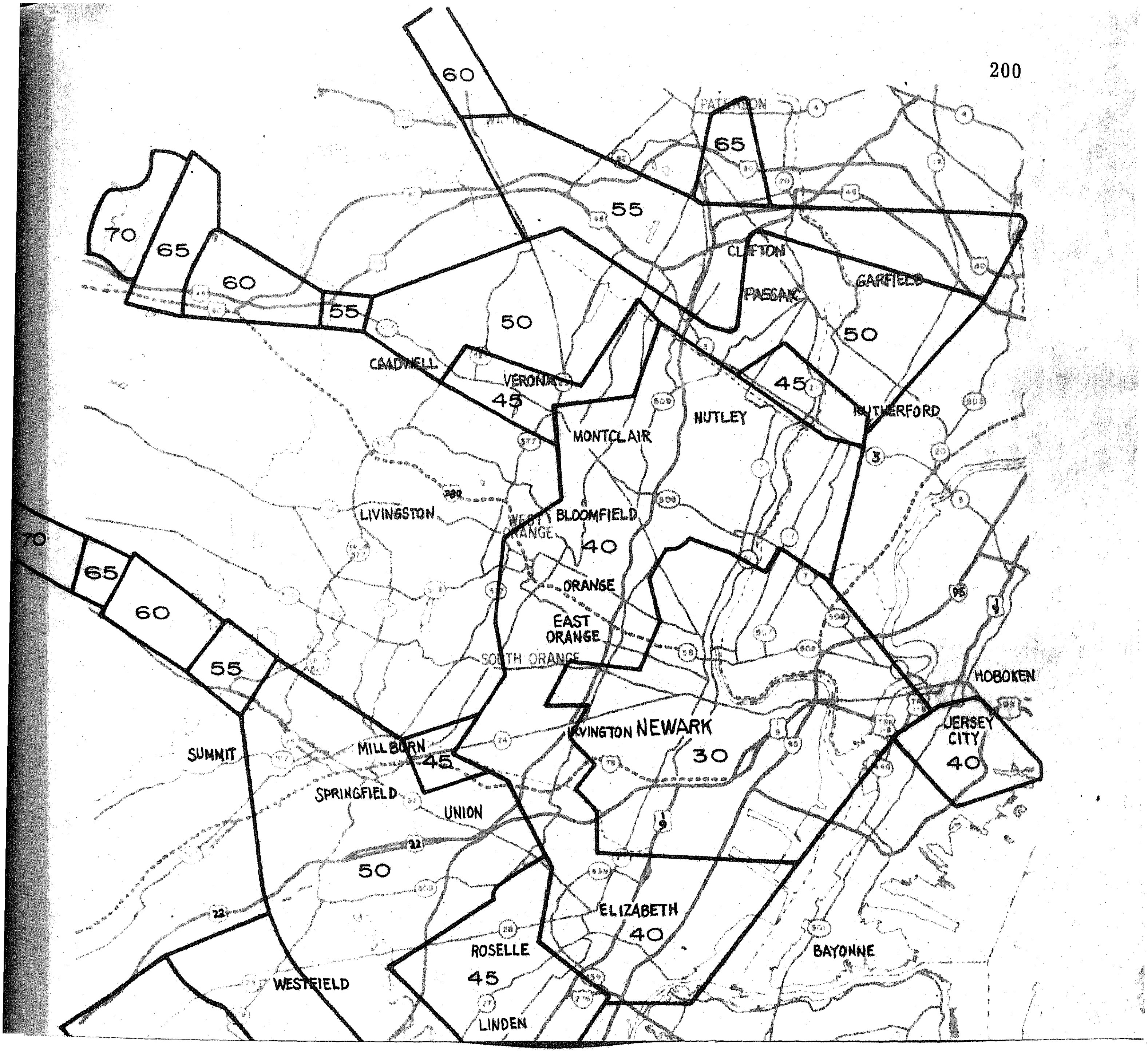
FIGURE 18



SUNDAY SCHEDULE VARIATION

LEGEND 13.8%.... % of Sunday Total
6.7 %... % of Average Weekday Total

FIGURE 19



* Source; "Public Transportation and Acess to Job Opportunities, Newark to selected Employment Centers," Edwards and Kelcey, Inc. August, 1970

FIGURE 20 BUS FARE STRUCTURE FROM CENTER OF NEWARK, N.J. (JULY, 1970)

APPENDIX E

COMPUTER INPUT AND OUTPUT

1. LEE·LPDGEN·INPUT
2. LEE·LPDGEN·DATA
3. LEE·LPTEST·INPUT
4. LEE·LPTEST·OUTPUT
5. LEE·LEEDP·INPUT
6. LPDGEN·INPUT·VERIFICATION
7. LINEAR PROGRAMMING OUTPUT (MPS/360)
8. DYNAMIC PROGRAMMING OUTPUT

1

```
-1 38 66 27 00 11
  11
  66
  14
45.0
30.0
 2  3  3  4  3  2  2  3  3  4
 4
 1
 2
 1
14
 2
 2  3
 2
10  8
 1
14
 3
 2  3  4
 2
10  7
 1
14
 4
 2  3  4  5
 2
10  6
 3
10  7  5
 1
14
 3
 3  4  5
 5
 9 12  8  4  5
 1
14
 2
 2  1
 1
14
 2
 4  5
 1
14
 4
13  3  2  1
 5
13  8 10 12  1
 1
14
 4
 4  3  2  1
```

```
  3
  7 10   1
  1
 14
  5
  5   4   3   2   1
  4
  5   7  10   1
  4
  6  10  12   1
  1
 14
  7
 13   5   4   3   2  12  11
  5
 13   4   8  10  11
  4
 13   6  10  11
  1
 14
 2.40
  2.27    0.682
  3.06    0.716
  4.33    1.044
  4.55    1.064
  3.22    0.800
 11.13    2.828
  8.69    2.102
  7.43    1.764
  9.16    1.946
  8.48    1.840
  6.70    1.498
  5.00    0.000
 10.00    0.000
120.0     0.000
   .40
```

-1 38 66 27 0 11
11 14 66

1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	1.00	1.00	1.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00
1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	1.00	1.00	1.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	1.00	1.00	0.00

0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00
1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	1.00	1.00	1.00	1.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	1.00	1.00	1.00	1.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00		
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.10
0.00	0.00	0.00	0.10	0.10	0.00	0.10	0.10
0.00	0.10	0.10	0.10	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.14	0.00	0.00	0.00	0.14	0.14	0.00	0.14
0.14	0.00	0.14	0.14	0.14	0.00	0.00	0.00
0.00	0.00	0.10	0.00	0.10	0.00	0.00	0.10
0.00	0.00	0.10	0.00	0.00	0.00	0.00	0.00
0.00	0.10	0.00	0.00	0.00	0.10	0.00	0.00
0.10	0.00	0.00	0.10	0.00	0.00	0.00	0.10
0.00	0.00	0.00	0.14	0.00	0.14	0.00	0.00
0.14	0.00	0.00	0.14	0.00	0.00	0.00	0.00
0.00	0.00	0.14	0.00	0.00	0.00	0.14	0.00

0.00	0.14	0.00	0.00	0.14	0.00	0.00	0.00
0.14	0.00	0.00	0.00	0.00	0.00	0.10	0.00
0.00	0.10	0.00	0.00	0.10	0.00	0.00	0.00
0.10	0.00	0.00	0.00	0.00	0.00	0.00	0.10
0.00	0.00	0.10	0.00	0.00	0.10	0.00	0.00
0.00	0.10	0.00	0.00	0.00	0.00	0.00	0.14
0.00	0.00	0.14	0.00	0.00	0.14	0.00	0.00
0.00	0.14	0.00	0.00	0.00	0.00	0.00	0.00
0.14	0.00	0.00	0.14	0.00	0.00	0.14	0.00
0.00	0.00	0.14	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.10	0.00	0.00	0.10	0.00
0.00	0.00	0.10	0.10	0.00	0.00	0.00	0.10
0.00	0.00	0.00	0.00	0.10	0.00	0.00	0.10
0.00	0.00	0.00	0.10	0.10	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.14	0.00	0.00	0.14
0.00	0.00	0.00	0.14	0.14	0.00	0.00	0.00
0.14	0.00	0.00	0.00	0.00	0.14	0.00	0.00
0.14	0.00	0.00	0.00	0.14	0.14	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.10	0.00	0.10	0.00	0.10	0.10	0.00	0.00
0.00	0.10	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.10	0.10	0.00	0.00	0.10	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.14	0.00	0.14	0.00	0.14	0.14	0.00
0.00	0.00	0.14	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.14	0.14	0.00	0.00	0.14	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.10	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.10	0.00	0.00
0.00	0.10	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.14	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.14	0.00
0.00	0.00	0.14	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.10	0.00	0.00	0.00	0.10	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.10	0.00	0.00	0.10	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.14	0.00	0.00	0.00	0.14
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.14	0.00	0.00	0.14
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.10	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.10	0.00	0.00	0.00	0.00
0.00	0.00	0.10	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.10	0.00	0.00	0.00
0.00	0.00	0.14	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.14	0.00	0.00	0.00
0.00	0.00	0.00	0.14	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.14	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.10	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.14	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.10	0.00	0.00
0.10	0.00	0.00	0.10	0.10	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.10	0.00
0.00	0.10	0.00	0.00	0.10	0.10	0.00	0.00
0.10	0.10	0.00	0.00	0.00	0.00	0.14	0.00
0.00	0.14	0.00	0.00	0.14	0.14	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.14
0.00	0.00	0.14	0.00	0.00	0.14	0.14	0.00
0.00	0.14	0.14	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.10	0.10	0.10	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.14	0.14	0.14	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00
0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00
0.00	1.00	0.00	1.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00
0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00
0.00	0.00	1.00	0.00	1.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	1.00	1.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	1.00	1.00	1.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	1.00	1.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00
1.00	0.00	0.00	1.00	0.00	0.00	1.00	0.00
0.00	1.00	0.00	0.00	0.00	1.00	0.00	0.00
1.00	0.00	1.00	0.00	1.00	0.00	0.00	1.00
0.00	0.00	1.00	0.00	0.00	0.00	1.00	0.00
0.00	0.00	1.00	0.00	1.00	0.00	0.00	1.00
0.00	0.00	1.00	0.00	0.00	0.00	1.00	0.00
0.00	1.00	0.00	1.00	0.00	1.00	0.00	0.00
1.00	0.00	0.00	1.00	0.00	0.00	0.00	1.00
0.00	0.00	0.00	1.00				
0.31	0.00	0.74	1.59	0.00	1.19	1.72	0.00
1.52	1.96	2.04	0.00	1.21	2.44	0.00	0.53
0.00	0.78	0.00	0.97	1.82	0.00	1.42	1.94
0.00	1.74	2.27	2.19	0.00	2.19	2.72	2.63
0.00	0.44	0.00	1.06	2.27	0.00	1.71	2.45
0.00	2.17	2.80	2.91	0.00	1.73	3.48	0.00
0.76	0.00	1.11	0.00	1.38	2.60	0.00	2.03

2.78	0.00	2.49	3.24	3.13	0.00	3.12	3.88
3.76	0.00						
0.07	0.00	0.18	0.36	0.00	0.28	0.39	0.00
0.36	0.47	0.47	0.00	0.29	0.56	0.00	0.14
0.00	0.19	0.00	0.24	0.43	0.00	0.35	0.46
0.00	0.43	0.34	0.53	0.00	0.51	0.62	0.62
0.00	0.10	0.30	0.25	0.51	0.00	0.40	0.56
0.00	0.52	0.57	0.68	0.00	0.42	0.30	0.00
0.20	0.00	0.27	0.00	0.35	0.61	0.00	0.50
0.66	0.00	0.62	0.77	0.76	0.00	0.73	0.88
0.88	0.00						
1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	1.00	1.00	1.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	1.00	1.00	1.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00
1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00
1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	1.00	1.00	1.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	1.00	1.00	1.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

0.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	1.00	1.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	1.00	1.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00
1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00
1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	1.00	1.00	1.00	1.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	1.00	1.00	1.00	1.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	1.00	1.00	1.00	1.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	1.00	1.00	1.00	1.00		
-0.21	4.40	0.07	0.60	4.40	0.36	0.68	4.40
0.57	0.85	0.89	4.40	0.37	1.33	4.40	-0.05
4.40	0.10	4.40	0.63	1.36	4.40	0.52	0.84
4.40	0.73	1.05	1.21	4.40	1.59	1.70	1.67
4.40	-0.13	4.40	0.15	0.75	4.40	0.48	0.85
4.40	0.72	1.05	1.09	4.40	0.50	1.57	4.40

0.01	4.40	0.18	4.40	0.74	1.54	4.40	0.67
1.04	4.40	0.91	1.28	1.44	4.40	1.81	1.97
1.93	4.40						

NCOST
LU001
LU002
LU003
LU004
LU005
LU006
LU007
LU008
LU009
LU010
LU011
LC001
LC002
LC003
LC004
LC005
LC006
LC007
LC008
LC009
LC010
LC011
LC012
LC013
LC014
LB001
LB002
GD001
GD002
GD003
GD004
GD005
GD006
GD007
GD008
GD009
GD010
GD011
CL01
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CM01
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CM33

4

```
**************************************************
12345678901234567890123456789012345678901234567890
 N   COST
 L   U001
 L   U002
 L   U003
 L   U004
 L   U005
 L   U006
 L   U007
 L   U008
 L   U009
 L   U010
 L   U011
 L   C001
 L   C002
 L   C003
 L   C004
 L   C005
 L   C006
 L   C007
 L   C008
 L   C009
 L   C010
 L   C011
 L   C012
 L   C013
 L   C014
 L   B001
 L   B002
 G   D001
 G   D002
 G   D003
 G   D004
 G   D005
 G   D006
 G   D007
 G   D008
 G   D009
 G   D010
 G   D011
**************************************************
    CL01      COST           -0.210
    CL01      U001            1.000
    CL01      C002            0.100
    CL01      B001            0.310
    CL01      B002            0.070
    CL01      D001            1.000
    CL02      COST            4.400
    CL02      U001            1.000
    CL02      C014            1.000
    CL02      D001            1.000
    CL03      COST            0.070
    CL03      U002            1.000
```

CL03	C002	0.100
CL03	C003	0.100
CL03	B001	0.740
CL03	B002	0.180
CL03	D002	1.000
CL04	COST	0.600
CL04	U002	1.000
CL04	C008	0.100
CL04	C010	0.100
CL04	B001	1.590
CL04	B002	0.360
CL04	D002	1.000
CL05	COST	4.400
CL05	U002	1.000
CL05	C014	1.000
CL05	D002	1.000
CL06	COST	0.360
CL06	U003	1.000
CL06	C002	0.100
CL06	C003	0.100
CL06	C004	0.100
CL06	B001	1.190
CL06	B002	0.280
CL06	D003	1.000
CL07	COST	0.680
CL07	U003	1.000
CL07	C007	0.100
CL07	C010	0.100
CL07	B001	1.720
CL07	B002	0.390
CL07	D003	1.000
CL08	COST	4.400
CL08	U003	1.000
CL08	C014	1.000
CL08	D003	1.000
CL09	COST	0.570
CL09	U004	1.000
CL09	C002	0.100
CL09	C003	0.100
CL09	C004	0.100
CL09	C005	0.100
CL09	B001	1.520
CL09	B002	0.360
CL09	D004	1.000
CL10	COST	0.850
CL10	U004	1.000
CL10	C006	0.100
CL10	C010	0.100
CL10	B001	1.960
CL10	B002	0.470
CL10	D004	1.000
CL11	COST	0.890
CL11	U004	1.000
CL11	C005	0.100

CL11	C007	0.100
CL11	C010	0.100
CL11	B001	2.040
CL11	B002	0.470
CL11	D004	1.000
CL12	COST	4.400
CL12	U004	1.000
CL12	C014	1.000
CL12	D004	1.000
CL13	COST	0.370
CL13	U005	1.000
CL13	C003	0.100
CL13	C004	0.100
CL13	C005	0.100
CL13	B001	1.210
CL13	B002	0.290
CL13	D005	1.000
CL14	COST	1.330
CL14	U005	1.000
CL14	C004	0.100
CL14	C005	0.100
CL14	C008	0.100
CL14	C009	0.100
CL14	C012	1.000
CL14	B001	2.440
CL14	B002	0.560
CL14	D005	1.000
CL15	COST	4.400
CL15	U005	1.000
CL15	C014	1.000
CL15	D005	1.000
CL16	COST	-0.050
CL16	U006	1.000
CL16	C001	0.100
CL16	C002	0.100
CL16	B001	0.530
CL16	B002	0.140
CL16	D006	1.000
CL17	COST	4.400
CL17	U006	1.000
CL17	C014	1.000
CL17	D006	1.000
CL18	COST	0.100
CL18	U007	1.000
CL18	C004	0.100
CL18	C005	0.100
CL18	B001	0.780
CL18	B002	0.190
CL18	D007	1.000
CL19	COST	4.400
CL19	U007	1.000
CL19	C014	1.000
CL19	D007	1.000
CL20	COST	0.630

CL20	U008	1.000
CL20	C001	0.100
CL20	C002	0.100
CL20	C003	0.100
CL20	C013	1.000
CL20	B001	0.970
CL20	B002	0.240
CL20	D008	1.000
CL21	COST	1.360
CL21	U008	1.000
CL21	C001	0.100
CL21	C008	0.100
CL21	C010	0.100
CL21	C012	1.000
CL21	C013	1.000
CL21	B001	1.820
CL21	B002	0.430
CL21	D008	1.000
CL22	COST	4.400
CL22	U008	1.000
CL22	C014	1.000
CL22	D008	1.000
CL23	COST	0.520
CL23	U009	1.000
CL23	C001	0.100
CL23	C002	0.100
CL23	C003	0.100
CL23	C004	0.100
CL23	B001	1.420
CL23	B002	0.350
CL23	D009	1.000
CL24	COST	0.840
CL24	U009	1.000
CL24	C001	0.100
CL24	C007	0.100
CL24	C010	0.100
CL24	B001	1.940
CL24	B002	0.460
CL24	D009	1.000
CL25	COST	4.400
CL25	U009	1.000
CL25	C014	1.000
CL25	D009	1.000
CL26	COST	0.730
CL26	U010	1.000
CL26	C001	0.100
CL26	C002	0.100
CL26	C003	0.100
CL26	C004	0.100
CL26	C005	0.100
CL26	B001	1.740
CL26	B002	0.430
CL26	D010	1.000
CL27	COST	1.050

CL27	U010	1.000
CL27	C001	0.100
CL27	C005	0.100
CL27	C007	0.100
CL27	C010	0.100
CL27	B001	2.270
CL27	B002	0.540
CL27	D010	1.000
CL28	COST	1.210
CL28	U010	1.000
CL28	C001	0.100
CL28	C006	0.100
CL28	C010	0.100
CL28	C012	1.000
CL28	B001	2.190
CL28	B002	0.530
CL28	D010	1.000
CL29	COST	4.400
CL29	U010	1.000
CL29	C014	1.000
CL29	D010	1.000
CL30	COST	1.590
CL30	U011	1.000
CL30	C002	0.100
CL30	C003	0.100
CL30	C004	0.100
CL30	C005	0.100
CL30	C011	0.100
CL30	C012	1.000
CL30	C013	1.000
CL30	B001	2.190
CL30	B002	0.510
CL30	D011	1.000
CL31	COST	1.700
CL31	U011	1.000
CL31	C004	0.100
CL31	C008	0.100
CL31	C010	0.100
CL31	C011	0.100
CL31	C013	1.000
CL31	B001	2.720
CL31	B002	0.620
CL31	D011	1.000
CL32	COST	1.670
CL32	U011	1.000
CL32	C006	0.100
CL32	C010	0.100
CL32	C011	0.100
CL32	C013	1.000
CL32	B001	2.630
CL32	B002	0.620
CL32	D011	1.000
CL33	COST	4.400
CL33	U011	1.000

CL33	C014	1.000
CL33	C011	1.000
CM01	COST	-0.180
CM01	C002	0.140
CM01	B001	0.440
CM01	B002	0.100
CM01	D001	1.000
CM02	COST	4.400
CM02	C014	1.000
CM02	D001	1.000
CM03	COST	0.150
CM03	C002	0.140
CM03	C003	0.140
CM03	B001	1.060
CM03	B002	0.250
CM03	D002	1.000
CM04	COST	0.750
CM04	C008	0.140
CM04	C010	0.140
CM04	B001	2.270
CM04	B002	0.510
CM04	D002	1.000
CM05	COST	4.400
CM05	C014	1.000
CM05	D002	1.000
CM06	COST	0.480
CM06	C002	0.140
CM06	C003	0.140
CM06	C004	0.140
CM06	B001	1.710
CM06	B002	0.400
CM06	D003	1.000
CM07	COST	0.850
CM07	C007	0.140
CM07	C010	0.140
CM07	B001	2.450
CM07	B002	0.560
CM07	D003	1.000
CM08	COST	4.400
CM08	C014	1.000
CM08	D003	1.000
CM09	COST	0.720
CM09	C002	0.140
CM09	C003	0.140
CM09	C004	0.140
CM09	C005	0.140
CM09	B001	2.170
CM09	B002	0.520
CM09	D004	1.000
CM10	COST	1.050
CM10	C006	0.140
CM10	C010	0.140
CM10	B001	2.800
CM10	B002	0.670

CM10	D004	1.000
CM11	COST	1.090
CM11	C005	0.140
CM11	C007	0.140
CM11	C010	0.140
CM11	B001	2.910
CM11	B002	0.680
CM11	D004	1.000
CM12	COST	4.400
CM12	C014	1.000
CM12	D004	1.000
CM13	COST	0.500
CM13	C003	0.140
CM13	C004	0.140
CM13	C005	0.140
CM13	B001	1.730
CM13	B002	0.420
CM13	D005	1.000
CM14	COST	1.570
CM14	C004	0.140
CM14	C005	0.140
CM14	C008	0.140
CM14	C009	0.140
CM14	C012	1.000
CM14	B001	3.480
CM14	B002	0.800
CM14	D005	1.000
CM15	COST	4.400
CM15	C014	1.000
CM15	D005	1.000
CM16	COST	0.010
CM16	C001	0.140
CM16	C002	0.140
CM16	B001	0.760
CM16	B002	0.200
CM16	D006	1.000
CM17	COST	4.400
CM17	C014	1.000
CM17	D006	1.000
CM18	COST	0.180
CM18	C004	0.140
CM18	C005	0.140
CM18	B001	1.110
CM18	B002	0.270
CM18	D007	1.000
CM19	COST	4.400
CM19	C014	1.000
CM19	D007	1.000
CM20	COST	0.740
CM20	C001	0.140
CM20	C002	0.140
CM20	C003	0.140
CM20	C013	1.000
CM20	B001	1.380

CM20	B002	0.350
CM20	D008	1.000
CM21	COST	1.540
CM21	C001	0.140
CM21	C008	0.140
CM21	C010	0.140
CM21	C012	1.000
CM21	C013	1.000
CM21	B001	2.600
CM21	B002	0.610
CM21	D008	1.000
CM22	COST	4.400
CM22	C014	1.000
CM22	D008	1.000
CM23	COST	0.670
CM23	C001	0.140
CM23	C002	0.140
CM23	C003	0.140
CM23	C004	0.140
CM23	B001	2.030
CM23	B002	0.500
CM23	D009	1.000
CM24	COST	1.040
CM24	C001	0.140
CM24	C007	0.140
CM24	C010	0.140
CM24	B001	2.780
CM24	B002	0.660
CM24	D009	1.000
CM25	COST	4.400
CM25	C014	1.000
CM25	D009	1.000
CM26	COST	0.910
CM26	C001	0.140
CM26	C002	0.140
CM26	C003	0.140
CM26	C004	0.140
CM26	C005	0.140
CM26	B001	2.490
CM26	B002	0.620
CM26	D010	1.000
CM27	COST	1.280
CM27	C001	0.140
CM27	C005	0.140
CM27	C007	0.140
CM27	C010	0.140
CM27	B001	3.240
CM27	B002	0.770
CM27	D010	1.000
CM28	COST	1.440
CM28	C001	0.140
CM28	C006	0.140
CM28	C010	0.140
CM28	C012	1.000

CM28	D001	3.130
CM28	D002	0.760
CM28	D010	1.000
CM29	COST	4.400
CM29	C014	1.000
CM29	D010	1.000
CM30	COST	1.810
CM30	C002	0.140
CM30	C003	0.140
CM30	C004	0.140
CM30	C005	0.140
CM30	C011	0.140
CM30	C012	1.000
CM30	C013	1.000
CM30	D001	3.120
CM30	D002	0.730
CM30	D011	1.000
CM31	COST	1.970
CM31	C004	0.140
CM31	C008	0.140
CM31	C010	0.140
CM31	C011	0.140
CM31	C013	1.000
CM31	D001	3.880
CM31	D002	0.880
CM31	D011	1.000
CM32	COST	1.930
CM32	C006	0.140
CM32	C010	0.140
CM32	C011	0.140
CM32	C013	1.000
CM32	D001	3.760
CM32	D002	0.880
CM32	D011	1.000
CM33	COST	4.400
CM33	C014	1.000
CM33	D011	1.000

****

RHS01	U001	458.000
RHS01	U002	1178.000
RHS01	U003	634.000
RHS01	U004	1135.000
RHS01	U005	330.000
RHS01	U006	755.000
RHS01	U007	624.000
RHS01	U008	1320.000
RHS01	U009	939.000
RHS01	U010	1249.000
RHS01	U011	424.000
RHS01	C001	87.000
RHS01	C002	87.000
RHS01	C003	87.000
RHS01	C004	87.000
RHS01	C005	87.000

RHSO1	C006	168.000
RHSO1	C007	0.000
RHSO1	C008	0.000
RHSO1	C009	0.000
RHSO1	C010	168.000
RHSO1	C011	168.000
RHSO1	C012	100000.000
RHSO1	C013	100000.000
RHSO1	C014	100000.000
RHSO1	B001	200000.000
RHSO1	B002	30000.000
RHSO1	D001	572.000
RHSO1	D002	1472.000
RHSO1	D003	793.000
RHSO1	D004	1418.000
RHSO1	D005	413.000
RHSO1	D006	674.000
RHSO1	D007	780.000
RHSO1	D008	1650.000
RHSO1	D009	1174.000
RHSO1	D010	1561.000
RHSO1	D011	530.000

```
*************************************************
```

5

```
        @P

 1.0000   3 50  5  6  9 10000.00
 2.0000  4.00 4.00 6.00
 3.0000 18.0027.0027.00
 4.0000   .70  .70  .80
 5.0000   .85  .95 1.00
 6.0000  1846500.00 1495250.00 1500000.00 1500000.00
 7.0000  1455750.00  904750.00  956000.00 1007250.00
 8.0000   228850.00  128900.00  146300.00  163700.00
 9.0000  6190500.00 5345500.00 4974250.00 5061250.00
10.0000  1192200.00 1098000.00 1021150.00  973600.00
11.0000  2287090.00 2083575.00 1970345.00 1880190.00
12.0000 SUN
13.0000 SAT OFF
14.0000 WEEK OFF
15.0000 SAT PEAK
16.0000 A.M. PEAK
17.0000 P.M. PEAK

18.
```

6

```
/DO LEE.PROC
%/PROC C
%/FILE LEE.LPDGEN.INPUT,LINK=DSET75,FCBTYPE=SAM,RECFORM=V
%/FILE LEE.LPDGEN.DATA,LINK=DSET77,FCBTYPE=SAM,RECFORM=V
%/EXEC LPDGEN
% C P500 LOADING.
 FORTRAN IV PROGRAM LPDGEN  STARTED --- 08/23/73

 DO YOU WISH TO VERIFY INPUT DATA (Y,N)?
*Y

 DATA VERIFICATION

 THERE ARE  11 DEMANDS
 AND  66 DEMAND-CHAINS

 DEMAND#  CHAIN#      LINKS
    1        1         2
    1        2        14
    2        1         2    3
    2        2        10    8
    2        3        14
    3        1         2    3    4
    3        2        10    7
    3        3        14
    4        1         2    3    4    5
    4        2        10    6
    4        3        10    7    5
    4        4        14
    5        1         3    4    5
    5        2         9   12    8    4    5
    5        3        14
    6        1         2    1
    6        2        14
    7        1         4    5
    7        2        14
    8        1        13    3    2    1
    8        2        13    8   10   12    1
    8        3        14
    9        1         4    3    2    1
    9        2         7   10    1
    9        3        14
   10        1         5    4    3    2    1
   10        2         5    7   10    1
   10        3         6   10   12    1
   10        4        14
   11        1        13    5    4    3    2   12   11
   11        2        13    4    8   10   11
   11        3        13    6   10   11
   11        4        14
```

```
THERE ARE  14 LINKS

LINK#  TRAVEL TIME  OPERATING COST
   1      2.270        0.682
   2      3.060        0.716
   3      4.330        1.044
   4      4.550        1.064
   5      3.220        0.800
   6     11.130        2.828
   7      8.690        2.102
   8      7.430        1.764
   9      9.160        1.946
  10      8.480        1.840
  11      6.700        1.498
  12      5.000        0.000
  13     10.000        0.000
  14    120.000        0.000

 LOAD FACTOR 1=  10.0
 LOAD FACTOR 2=   7.0
 PASSENGER COST=  2.40
 FARE=  0.40


 IS INPUT DATA CORRECT (Y,N)?
*Y
 **FORTRAN ** STOP
%/PRINT LEE.LPDGEN.DATA
% C SB01 PRINT LEE.LPDGEN.DATA INITIATED: TSN=7580.
%/ENDP
/
```

CONTROL PROGRAM COMPILER - MPS/360 V2-M8

7

```
0001          PROGRAM
0002          INITIALZ
0064          MOVE(XDATA,'LPTEST')
0065          MOVE(XPBNAME,'PBFILE')
0066          CONVERT('SUMMARY')
0067          BCDOUT
0068          SETUP
0069          MOVE(XOBJ,'COST')
0070          MOVE(XRHS,'RHS01')
0071          PRIMAL
0072          TRACE
0073          SOLUTION
0074          EXIT
0075          PEND
```

```
                EXECUTOR.    MPS/360 V2-M8

CONVERT LPTEST    TO PBFILE

TIME =    0.02

    SUMMARY

1- ROWS SECTION.

   0 MINOR ERROR(S) -    0 MAJOR ERROR(S).

2- COLUMNS SECTION.

   0 MINOR ERROR(S) -    0 MAJOR ERROR(S).

3- RHS'S SECTION.

  RHS01

   0 MINOR ERROR(S) -    0 MAJOR ERROR(S).
```

NUMBER OF ELEMENTS BY COLUMN ORDER

40	CL01	6	CL02	4	CL03	7	CL04	7	CL05	4	CL06	8	CL07	7
47	CL08	4	CL09	9	CL10	7	CL11	8	CL12	4	CL13	8	CL14	10
54	CL15	4	CL16	7	CL17	4	CL18	7	CL19	4	CL20	9	CL21	10
61	CL22	4	CL23	9	CL24	8	CL25	4	CL26	10	CL27	9	CL28	9
68	CL29	4	CL30	12	CL31	10	CL32	9	CL33	4	CM01	5	CM02	3
75	CM03	6	CM04	6	CM05	3	CM06	7	CM07	6	CM08	3	CM09	8
82	CM10	6	CM11	7	CM12	3	CM13	7	CM14	9	CM15	3	CM16	6
89	CM17	3	CM18	6	CM19	3	CM20	8	CM21	9	CM22	3	CM23	8
96	CM24	7	CM25	3	CM26	9	CM27	8	CM28	8	CM29	3	CM30	11
103	CM31	9	CM32	8	CM33	3								

NUMBER OF ELEMENTS BY ROW ORDER, EXCLUDING RHS'S, INCLUDING SLACK ELEMENT

1	N COST	67	L U001	3	L U002	4	L U003	4	L U004	5	L U005	4	L U006	3
8	L U007	3	L U008	4	L U009	4	L U010	5	L U011	5	L C001	17	L C002	19
15	L C003	17	L C004	19	L C005	17	L C006	7	L C007	9	L C008	9	L C009	3
22	L C010	21	L C011	7	L C012	9	L C013	11	L C014	23	L B001	45	L B002	45
29	G D001	5	G D002	7	G D003	7	G D004	9	G D005	7	G D006	5	G D007	5
36	G D008	7	G D009	7	G D010	9	G D011	9						

PROBLEM STATISTICS -   39 ROWS,   105 VARIABLES,   466 ELEMENTS, DENSITY = 11.37

THESE STATISTICS INCLUDE ONE SLACK VARIABLE FOR EACH ROW.

0 MINOR ERRORS,   0 MAJOR ERRORS.

```
NAME          LPTEST
ROWS
 N  COST
 L  U001
 L  U002
 L  U003
 L  U004
 L  U005
 L  U006
 L  U007
 L  U008
 L  U009
 L  U010
 L  U011
 L  C001
 L  C002
 L  C003
 L  C004
 L  C005
 L  C006
 L  C007
 L  C008
 L  C009
 L  C010
 L  C011
 L  C012
 L  C013
 L  C014
 L  B001
 L  B002
 G  D001
 G  D002
 G  D003
 G  D004
 G  D005
 G  D006
 G  D007
 G  D008
 G  D009
 G  D010
 G  D011
COLUMNS
    CL01      COST      -     .21000   U001          1.00000
    CL01      C002            .10000   B001           .31000
    CL01      B002            .07000   D001          1.00000
    CL02      COST           4.40000   U001          1.00000
    CL02      C014           1.00000   D001          1.00000
    CL03      COST            .07000   U002          1.00000
    CL03      C002            .10000   C003           .10000
    CL03      B001            .74000   B002           .18000
    CL03      D002           1.00000
    CL04      COST            .60000   U002          1.00000
    CL04      C008            .10000   C010           .10000
    CL04      B001           1.59000   B002           .36000
```

CL04	D002	1.00000		
CL05	COST	4.40000	U002	1.00000
CL05	C014	1.00000	D002	1.00000
CL06	COST	.36000	U003	1.00000
CL06	C002	.10000	C003	.10000
CL06	C004	.10000	B001	1.19000
CL06	B002	.28000	D003	1.00000
CL07	COST	.68000	U003	1.00000
CL07	C007	.10000	C010	.10000
CL07	B001	1.72000	B002	.39000
CL07	D003	1.00000		
CL08	COST	4.40000	U003	1.00000
CL08	C014	1.00000	D003	1.00000
CL09	COST	.57000	U004	1.00000
CL09	C002	.10000	C003	.10000
CL09	C004	.10000	C005	.10000
CL09	B001	1.52000	B002	.36000
CL09	D004	1.00000		
CL10	COST	.85000	U004	1.00000
CL10	C006	.10000	C010	.10000
CL10	B001	1.96000	B002	.47000
CL10	D004	1.00000		
CL11	COST	.89000	U004	1.00000
CL11	C005	.10000	C007	.10000
CL11	C010	.10000	B001	2.04000
CL11	B002	.47000	D004	1.00000
CL12	COST	4.40000	U004	1.00000
CL12	C014	1.00000	D004	1.00000
CL13	COST	.37000	U005	1.00000
CL13	C003	.10000	C004	.10000
CL13	C005	.10000	B001	1.21000
CL13	B002	.29000	D005	1.00000
CL14	COST	1.33000	U005	1.00000
CL14	C004	.10000	C005	.10000
CL14	C008	.10000	C009	.10000
CL14	C012	1.00000	B001	2.44000
CL14	B002	.56000	D005	1.00000
CL15	COST	4.40000	U005	1.00000
CL15	C014	1.00000	D005	1.00000
CL16	COST	- .05000	U006	1.00000
CL16	C001	.10000	C002	.10000
CL16	B001	.53000	B002	.14000
CL16	D006	1.00000		
CL17	COST	4.40000	U006	1.00000
CL17	C014	1.00000	D006	1.00000
CL18	COST	.10000	U007	1.00000
CL18	C004	.10000	C005	.10000
CL18	B001	.78000	B002	.19000
CL18	D007	1.00000		
CL19	COST	4.40000	U007	1.00000
CL19	C014	1.00000	D007	1.00000
CL20	COST	.63000	U008	1.00000
CL20	C001	.10000	C002	.10000
CL20	C003	.10000	C013	1.00000

CL20	B001	.97000	B002	.24000
CL20	D008	1.00000		
CL21	COST	1.36000	U008	1.00000
CL21	C001	.10000	C008	.10000
CL21	C010	.10000	C012	1.00000
CL21	C013	1.00000	B001	1.82000
CL21	B002	.43000	D008	1.00000
CL22	COST	4.40000	U008	1.00000
CL22	C014	1.00000	D008	1.00000
CL23	COST	.52000	U009	1.00000
CL23	C001	.10000	C002	.10000
CL23	C003	.10000	C004	.10000
CL23	B001	1.42000	B002	.35000
CL23	D009	1.00000		
CL24	COST	.84000	U009	1.00000
CL24	C001	.10000	C007	.10000
CL24	C010	.10000	B001	1.94000
CL24	B002	.46000	D009	1.00000
CL25	COST	4.40000	U009	1.00000
CL25	C014	1.00000	D009	1.00000
CL26	COST	.73000	U010	1.00000
CL26	C001	.10000	C002	.10000
CL26	C003	.10000	C004	.10000
CL26	C005	.10000	B001	1.74000
CL26	B002	.43000	D010	1.00000
CL27	COST	1.05000	U010	1.00000
CL27	C001	.10000	C005	.10000
CL27	C007	.10000	C010	.10000
CL27	B001	2.27000	B002	.54000
CL27	D010	1.00000		
CL28	COST	1.21000	U010	1.00000
CL28	C001	.10000	C006	.10000
CL28	C010	.10000	C012	1.00000
CL28	B001	2.19000	B002	.53000
CL28	D010	1.00000		
CL29	COST	4.40000	U010	1.00000
CL29	C014	1.00000	D010	1.00000
CL30	COST	1.59000	U011	1.00000
CL30	C002	.10000	C003	.10000
CL30	C004	.10000	C005	.10000
CL30	C011	.10000	C012	1.00000
CL30	C013	1.00000	B001	2.19000
CL30	B002	.51000	D011	1.00000
CL31	COST	1.70000	U011	1.00000
CL31	C004	.10000	C008	.10000
CL31	C010	.10000	C011	.10000
CL31	C013	1.00000	B001	2.72000
CL31	B002	.62000	D011	1.00000
CL32	COST	1.67000	U011	1.00000
CL32	C006	.10000	C010	.10000
CL32	C011	.10000	C013	1.00000
CL32	B001	2.63000	B002	.62000
CL32	D011	1.00000		
CL33	COST	4.40000	U011	1.00000

CL33	C014	1.00000	D011	1.00000
CM01	COST	- .18000	C002	.14000
CM01	B001	.44000	B002	.10000
CM01	D001	1.00000		
CM02	COST	4.40000	C014	1.00000
CM02	D001	1.00000		
CM03	COST	.15000	C002	.14000
CM03	C003	.14000	B001	1.06000
CM03	B002	.25000	D002	1.00000
CM04	COST	.75000	C008	.14000
CM04	C010	.14000	B001	2.27000
CM04	B002	.51000	D002	1.00000
CM05	COST	4.40000	C014	1.00000
CM05	D002	1.00000		
CM06	COST	.48000	C002	.14000
CM06	C003	.14000	C004	.14000
CM06	B001	1.71000	B002	.40000
CM06	D003	1.00000		
CM07	COST	.85000	C007	.14000
CM07	C010	.14000	B001	2.45000
CM07	B002	.56000	D003	1.00000
CM08	COST	4.40000	C014	1.00000
CM08	D003	1.00000		
CM09	COST	.72000	C002	.14000
CM09	C003	.14000	C004	.14000
CM09	C005	.14000	B001	2.17000
CM09	B002	.52000	D004	1.00000
CM10	COST	1.05000	C006	.14000
CM10	C010	.14000	B001	2.80000
CM10	B002	.67000	D004	1.00000
CM11	COST	1.09000	C005	.14000
CM11	C007	.14000	C010	.14000
CM11	B001	2.91000	B002	.68000
CM11	D004	1.00000		
CM12	COST	4.40000	C014	1.00000
CM12	D004	1.00000		
CM13	COST	.50000	C003	.14000
CM13	C004	.14000	C005	.14000
CM13	B001	1.73000	B002	.42000
CM13	D005	1.00000		
CM14	COST	1.57000	C004	.14000
CM14	C005	.14000	C008	.14000
CM14	C009	.14000	C012	1.00000
CM14	B001	3.48000	B002	.80000
CM14	D005	1.00000		
CM15	COST	4.40000	C014	1.00000
CM15	D005	1.00000		
CM16	COST	.01000	C001	.14000
CM16	C002	.14000	B001	.76000
CM16	B002	.20000	D006	1.00000
CM17	COST	4.40000	C014	1.00000
CM17	D006	1.00000		
CM18	COST	.18000	C004	.14000
CM18	C005	.14000	B001	1.11000

CM18	B002	.27000	D007	1.00000
CM19	COST	4.40000	C014	1.00000
CM19	D007	1.00000		
CM20	COST	.74000	C001	.14000
CM20	C002	.14000	C003	.14000
CM20	C013	1.00000	B001	1.38000
CM20	B002	.35000	D008	1.00000
CM21	COST	1.54000	C001	.14000
CM21	C008	.14000	C010	.14000
CM21	C012	1.00000	C013	1.00000
CM21	B001	2.60000	B002	.61000
CM21	D008	1.00000		
CM22	COST	4.40000	C014	1.00000
CM22	D008	1.00000		
CM23	COST	.67000	C001	.14000
CM23	C002	.14000	C003	.14000
CM23	C004	.14000	B001	2.03000
CM23	B002	.50000	D009	1.00000
CM24	COST	1.04000	C001	.14000
CM24	C007	.14000	C010	.14000
CM24	B001	2.78000	B002	.66000
CM24	D009	1.00000		
CM25	COST	4.40000	C014	1.00000
CM25	D009	1.00000		
CM26	COST	.91000	C001	.14000
CM26	C002	.14000	C003	.14000
CM26	C004	.14000	C005	.14000
CM26	B001	2.49000	B002	.62000
CM26	D010	1.00000		
CM27	COST	1.28000	C001	.14000
CM27	C005	.14000	C007	.14000
CM27	C010	.14000	B001	3.24000
CM27	B002	.77000	D010	1.00000
CM28	COST	1.44000	C001	.14000
CM28	C006	.14000	C010	.14000
CM28	C012	1.00000	B001	3.13000
CM28	B002	.76000	D010	1.00000
CM29	COST	4.40000	C014	1.00000
CM29	D010	1.00000		
CM30	COST	1.81000	C002	.14000
CM30	C003	.14000	C004	.14000
CM30	C005	.14000	C011	.14000
CM30	C012	1.00000	C013	1.00000
CM30	B001	3.12000	B002	.73000
CM30	D011	1.00000		
CM31	COST	1.97000	C004	.14000
CM31	C008	.14000	C010	.14000
CM31	C011	.14000	C013	1.00000
CM31	B001	3.88000	B002	.88000
CM31	D011	1.00000		
CM32	COST	1.93000	C006	.14000
CM32	C010	.14000	C011	.14000
CM32	C013	1.00000	B001	3.76000
CM32	B002	.88000	D011	1.00000

```
    CM33       COST          4.40000   C014          1.00000
    CM33       D011          1.00000
RHS
    RHS01      U001        458.00000   U002       1178.00000
    RHS01      U003        634.00000   U004       1135.00000
    RHS01      U005        330.00000   U006        755.00000
    RHS01      U007        624.00000   U008       1320.00000
    RHS01      U009        939.00000   U010       1249.00000
    RHS01      U011        424.00000   C001         87.00000
    RHS01      C002         87.00000   C003         87.00000
    RHS01      C004         87.00000   C005         87.00000
    RHS01      C006        168.00000   C010        168.00000
    RHS01      C011        168.00000   C012       100000.0000
    RHS01      C013       100000.0000  C014       100000.0000
    RHS01      B001       200000.0000  B002       30000.00000
    RHS01      D001        572.00000   D002       1472.00000
    RHS01      D003        793.00000   D004       1418.00000
    RHS01      D005        413.00000   D006        674.00000
    RHS01      D007        780.00000   D008       1650.00000
    RHS01      D009       1174.00000   D010       1561.00000
    RHS01      D011        530.00000
ENDATA
```

```
                EXECUTOR.     MPS/360 V2-M8

SETUP   PBFILE

TIME =    0.19

 MATRIX1  ASSIGNED TO MATRIX1

 ETA1     ASSIGNED TO ETA1

 SCRATCH1 ASSIGNED TO SCRATCH1
 SCRATCH2 ASSIGNED TO SCRATCH2

 MAXIMUM PRICING NOT REQUIRED - MAXIMUM POSSIBLE    7

 NO CYCLING
```

POOLS	NUMBER	SIZE	CORE
H.REG-BITS MAP			168
WORK REGIONS	9	336	3024
MATRIX BUFFERS	2	3400	6800
ETA BUFFERS	4	7152	28608

	TOTAL	NORMAL	.FREE.	FIXED	BOUNDED
ROWS (LOG.VAR.)	39	38	1	0	0
COLUMNS (STR.VAR.)	66	66	0	0	0

```
      466 ELEMENTS - DENSITY =  11.37 -   2 MATRIX RECORDS (WITHOUT RHS'S)

PRIMAL        OBJ = COST         RHS = RHS01

TIME =     0.25  MINS.       PRICING    7
 SCALE =       .
```

	ITER NUMBER	NUMBER INFEAS	VECTOR OUT	VECTOR IN	REDUCED COST	SUM INFEAS
M	1	10	26	87	1.00000-	10624.0
	2		33	105	1.00000-	10094.0
M	3	8	39	74	1.00000-	9522.00
	4		29	89	1.00000-	8848.00
M	5	6	34	91	1.00000-	8068.00
	6		4	47	1.00000-	7434.00
	7		19	79	1.00000-	7434.00
M	8	5	35	80	1.00000-	7275.00
	9		31	97	1.00000-	6101.00
	10		5	51	1.00000-	4966.00
M	11	3	37	84	1.00000-	4683.00
	12		32	77	1.00000-	3211.00
M	13	1	30	101	1.00000-	1650.00
M	14	0	38	94	1.00000-	.

```
 FEASIBLE SOLUTION

PRIMAL        OBJ = COST         RHS = RHS01

TIME =     0.26  MINS.       PRICING    7
```

EXECUTOR.   MPS/360 V2-M8

SCALE =   .
SCALE RESET TO   1.00000

	ITER NUMBER	NUMBER NONOPT	VECTOR OUT	VECTOR IN	REDUCED COST	FUNCTION VALUE
M	15	5	36	26	4.40000-	48562.8
	16		89	55	4.45000-	45563.5
	17		14	40	4.61000-	44659.9
M	18	24	51	49	3.55000-	40630.7
	19		8	57	4.30000-	37947.5
	20		17	52	4.03000-	36956.1
	21		20	43	3.80000-	36956.1
M	22	9	12	71	2.73000-	35798.6
	23		22	67	3.19000-	35412.6
	24		79	46	1.18429-	35412.6
M	25	4	2	56	.16000-	35370.7
	26		13	12	.46000-	35215.7
M	27	2	71	42	.34000-	35186.1

OPTIMAL SOLUTION

EXECUTOR. MPS/360 V2-M8

SOLUTION (OPTIMAL)

TIME = 0.27 MINS. ITERATION NUMBER = 27

...NAME...	...ACTIVITY...	DEFINED AS
FUNCTIONAL	35186.08000	COST
RESTRAINTS		RHS01

SECTION 1 - ROWS

	NUMBER	...ROW..	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
	1	COST	BS	35186.08000	35186.08000-	NONE	NONE	1.00000
	2	U001	UL	458.00000	.	NONE	458.00000	.28000
	3	U002	BS	87.00000	1091.00000	NONE	1178.00000	.
A	4	U003	UL	634.00000	.	NONE	634.00000	.
	5	U004	UL	1135.00000	.	NONE	1135.00000	.48000
	6	U005	BS	246.00000	84.00000	NONE	330.00000	.
	7	U006	BS	674.00000	81.00000	NONE	755.00000	.
	8	U007	UL	624.00000	.	NONE	624.00000	.27000
	9	U008	BS	.	1320.00000	NONE	1320.00000	.
	10	U009	BS	.	939.00000	NONE	939.00000	.
	11	U010	BS	545.00000	704.00000	NONE	1249.00000	.
	12	U011	BS	.	424.00000	NONE	424.00000	.
	13	C001	UL	87.00000	.	NONE	87.00000	1.20000
	14	C002	UL	87.00000	.	NONE	87.00000	43.30000
	15	C003	BS	33.30000	53.70000	NONE	87.00000	.
	16	C004	BS	87.00000	.	NONE	87.00000	.
	17	C005	UL	87.00000	.	NONE	87.00000	40.30000
	18	C006	BS	168.00000	.	NONE	168.00000	.
	19	C007	UL	.	.	NONE	.	6.50000
	20	C008	UL	.	.	NONE	.	7.30000
	21	C009	BS	.	.	NONE	.	.
	22	C010	UL	168.00000	.	NONE	168.00000	30.70000
	23	C011	BS	.	168.00000	NONE	168.00000	.
	24	C012	BS	545.00000	99455.00000	NONE	100000.00000	.
	25	C013	BS	.	100000.00000	NONE	100000.00000	.
	26	C014	BS	7617.00000	92383.00000	NONE	100000.00000	.
	27	B001	BS	4581.14000	195418.86000	NONE	200000.00000	.
	28	B002	BS	1105.42000	28894.58000	NONE	30000.00000	.
	29	D001	LL	572.00000	.	572.00000	NONE	4.40000-
	30	D002	LL	1472.00000	.	1472.00000	NONE	4.40000-
	31	D003	LL	793.00000	.	793.00000	NONE	4.40000-
	32	D004	LL	1418.00000	.	1418.00000	NONE	4.40000-
	33	D005	LL	413.00000	.	413.00000	NONE	4.40000-
	34	D006	LL	674.00000	.	674.00000	NONE	4.40000-
	35	D007	LL	780.00000	.	780.00000	NONE	4.40000-
	36	D008	LL	1650.00000	.	1650.00000	NONE	4.40000-
	37	D009	LL	1174.00000	.	1174.00000	NONE	4.40000-
	38	D010	LL	1561.00000	.	1561.00000	NONE	4.40000-
	39	D011	LL	530.00000	.	530.00000	NONE	4.40000-

SECTION 2 - COLUMNS

	NUMBER	.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
	40	CL01	BS	458.00000	.21000-	.	NONE	.
	41	CL02	LL	.	4.40000	.	NONE	.28000
	42	CL03	BS	87.00000	.07000	.	NONE	.
	43	CL04	BS	.	.60000	.	NONE	.
A	44	CL05	LL	.	4.40000	.	NONE	.
	45	CL06	LL	.	.36000	.	NONE	.29000
	46	CL07	BS	.	.68000	.	NONE	.
	47	CL08	BS	634.00000	4.40000	.	NONE	.
	48	CL09	LL	.	.57000	.	NONE	5.01000
	49	CL10	BS	1135.00000	.85000	.	NONE	.
	50	CL11	LL	.	.89000	.	NONE	4.72000
	51	CL12	LL	.	4.40000	.	NONE	.48000
	52	CL13	BS	246.00000	.37000	.	NONE	.
	53	CL14	LL	.	1.33000	.	NONE	1.69000
A	54	CL15	LL	.	4.40000	.	NONE	.
	55	CL16	BS	325.00000	.05000-	.	NONE	.
	56	CL17	BS	349.00000	4.40000	.	NONE	.
	57	CL18	BS	624.00000	.10000	.	NONE	.
	58	CL19	LL	.	4.40000	.	NONE	.27000
	59	CL20	LL	.	.63000	.	NONE	.68000
	60	CL21	LL	.	1.36000	.	NONE	.88000
A	61	CL22	LL	.	4.40000	.	NONE	.
	62	CL23	LL	.	.52000	.	NONE	.57000
	63	CL24	LL	.	.84000	.	NONE	.28000
A	64	CL25	LL	.	4.40000	.	NONE	.
	65	CL26	LL	.	.73000	.	NONE	4.81000
	66	CL27	LL	.	1.05000	.	NONE	4.52000
	67	CL28	BS	545.00000	1.21000	.	NONE	.
A	68	CL29	LL	.	4.40000	.	NONE	.
	69	CL30	LL	.	1.59000	.	NONE	5.55000
	70	CL31	LL	.	1.70000	.	NONE	1.10000
	71	CL32	LL	.	1.67000	.	NONE	.34000
A	72	CL33	LL	.	4.40000	.	NONE	.
	73	CM01	LL	.	.18000-	.	NONE	1.48200
	74	CM02	BS	114.00000	4.40000	.	NONE	.
	75	CM03	LL	.	.15000	.	NONE	1.81200
	76	CM04	LL	.	.75000	.	NONE	1.67000
	77	CM05	BS	1385.00000	4.40000	.	NONE	.
	78	CM06	LL	.	.48000	.	NONE	2.14200
	79	CM07	LL	.	.85000	.	NONE	1.65800
	80	CM08	BS	159.00000	4.40000	.	NONE	.
	81	CM09	LL	.	.72000	.	NONE	8.02400
	82	CM10	LL	.	1.05000	.	NONE	.94800
	83	CM11	LL	.	1.09000	.	NONE	7.54000
	84	CM12	BS	283.00000	4.40000	.	NONE	.
	85	CM13	LL	.	.50000	.	NONE	1.74200
	86	CM14	LL	.	1.57000	.	NONE	3.83400
	87	CM15	BS	167.00000	4.40000	.	NONE	.
	88	CM16	LL	.	.01000	.	NONE	1.84000

NUMBER	.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
A 89	CM17	LL	.	4.40000	.	NONE	.
90	CM18	LL	.	.18000	.	NONE	1.42200
91	CM19	BS	156.00000	4.40000	.	NONE	.
92	CM20	LL	.	.74000	.	NONE	2.57000
93	CM21	LL	.	1.54000	.	NONE	2.62800
94	CM22	BS	1650.00000	4.40000	.	NONE	.
95	CM23	LL	.	.67000	.	NONE	2.50000
96	CM24	LL	.	1.04000	.	NONE	2.01600
97	CM25	BS	1174.00000	4.40000	.	NONE	.
98	CM26	LL	.	.91000	.	NONE	8.38200
99	CM27	LL	.	1.28000	.	NONE	7.89800
100	CM28	LL	.	1.44000	.	NONE	1.50600
101	CM29	BS	1016.00000	4.40000	.	NONE	.
102	CM30	LL	.	1.81000	.	NONE	9.11400
103	CM31	LL	.	1.97000	.	NONE	2.89000
104	CM32	LL	.	1.93000	.	NONE	1.82800
105	CM33	BS	530.00000	4.40000	.	NONE	.

8

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 DO LEE.LEEDP.PROC
Z/PROC C
Z/FILE LEE.LEEDP.INPUT,LINK=DSET70,FCBTYPE=ISAM,RECFORM=V
Z/EXEC LEEDP
Z C P500 LOADING.
 FORTRAN IV PROGRAM LEEDP   STARTED --- 09/15/73

                        DYNAMIC PROGRAMMING MODEL

                                YOUNG LEE
          INPUT DATA

 M=  4                 DELTA= 50
 THETA=  5             NSTAGE=  6
 NSTATE=  9            OWC= $  5000.00
 STAGE    DELTA/THETA   STAGE FACTOR
   6         4.00          0.70
   5         4.00          0.70
   4         6.00          0.80
   3        18.00          0.85
   2        27.00          0.95
   1        27.00          1.00


                       BUS FREQUENCY COSTS
 STATE/STAGE 6          5        4         3         2         1
   1       1846500. 1455750.  228850. 6190500. 1192200. 2287090.
   2       1495250.  904750.  128900. 5345500. 1098000. 2083575.
   3       1500000.  956000.  146300. 4974250. 1021150. 1970345.
   4       1500000. 1007250.  163700. 5061250.  973600. 1880190.

 FLEETSIZE(  1,  6) HAS RANGE OF  1
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FLEETSIZE(  8,   2) HAS RANGE OF  4
FLEETSIZE(  9,   2) HAS RANGE OF  4
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FLEETSIZE(  2,   1) HAS RANGE OF  3
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FLEETSIZE(  4,   1) HAS RANGE OF  4
FLEETSIZE(  5,   1) HAS RANGE OF  4
FLEETSIZE(  6,   1) HAS RANGE OF  4
FLEETSIZE(  7,   1) HAS RANGE OF  4
FLEETSIZE(  8,   1) HAS RANGE OF  4
FLEETSIZE(  9,   1) HAS RANGE OF  4
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BUS FLEETSIZE COST MATRIX

STATE/STAGE	6	5	4	3	2	1
1	1846500.	1455750.	228850.	6190500.	1192200.	2287090.
2	1846500.	1455750.	228850.	5345500.	1021150.	1970345.
3	1846500.	1455750.	128900.	4974250.	973600.	1880190.
4	1495250.	904750.	128900.	4974250.	973600.	1880190.
5	1495250.	904750.	128900.	4974250.	973600.	1880190.
6	1495250.	904750.	128900.	4974250.	973600.	1880190.
7	1495250.	904750.	128900.	4974250.	973600.	1880190.
8	1495250.	904750.	128900.	4974250.	973600.	1880190.
9	1495250.	904750.	128900.	4974250.	973600.	1880190.

```
                 CALCULATION ======>


          PATH MATRIX
STATE/STAGE  1  2  3  4  5
  1          3  3  3  4  4
  2          3  3  3  4  4
  3          3  3  3  4  4
  4          4  4  4  4  4
  5          5  5  5  5  5
  6          6  6  6  6  6
  7          7  7  7  7  7
  8          8  8  8  8  8
  9          9  9  9  9  9


* * * * * * * * * * * BUS SYSTEM STUDY RESULTS * * * * * * * * * * *


          SCHEDULE PERIOD FLEET SIZE
SUN                   10 BUSES
SAT OFF               10 BUSES
WEEK OFF    @         10 BUSES
SAT PEAK              10 BUSES
A.M. PEAK             15 BUSES
P.M. PEAK             15 BUSES


SCHEDULE PERIOD SERVICE FREQUENCY
SUN                 150 DISPATCHES
SAT OFF             150 DISPATCHES
WEEK OFF            100 DISPATCHES
SAT PEAK             50 DISPATCHES
A.M. PEAK            50 DISPATCHES
P.M. PEAK            50 DISPATCHES


OPTIMUM FLEET SIZE OF PROPOSED BUS ROUTE IS    15 BUSES

OPTIMUM TOTAL BUS TRANSIT SYSTEM COST IS $    10424439.00


**FORTRAN ** STOP
%/ENDP
```

```
/DO LEE.LEEDP.PROC
%/PROC C
%/FILE LEE.LEEDP.INPUT,LINK=DSET70,FCBTYPE=ISAM,RECFORM=V
%/EXEC LEEDP
% C P500 LOADING.
 FORTRAN IV PROGRAM LEEDP   STARTED --- 09/15/73

                          DYNAMIC PROGRAMMING MODEL

                                    YOUNG LEE
           INPUT DATA

 M=  4                    DELTA= 50
 THETA=  5                NSTAGE=  6
 NSTATE=  9               OWC= $500000.00
 STAGE    DELTA/THETA    STAGE FACTOR
   6         4.00            0.70
   5         4.00            0.70
   4         6.00            0.80
   3        18.00            0.85
   2        27.00            0.95
   1        27.00            1.00


                          BUS FREQUENCY COSTS
 STATE/STAGE 6          5          4          3          2          1
   1      1846500. 1455750.  228850. 6190500. 1192200. 2287090.
   2      1495250.  904750.  128900. 5345500. 1098000. 2083575.
   3      1500000.  956000.  146300. 4974250. 1021150. 1970345.
   4      1500000. 1007250.  163700. 5061250.  973600. 1880190.

 FLEETSIZE(  1,   6) HAS RANGE OF  1
 FLEETSIZE(  2,   6) HAS RANGE OF  1
 FLEETSIZE(  3,   6) HAS RANGE OF  1
 FLEETSIZE(  4,   6) HAS RANGE OF  2
 FLEETSIZE(  5,   6) HAS RANGE OF  2
 FLEETSIZE(  6,   6) HAS RANGE OF  3
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 FLEETSIZE(  8,   6) HAS RANGE OF  3
 FLEETSIZE(  9,   6) HAS RANGE OF  4
 FLEETSIZE(  1,   5) HAS RANGE OF  1
 FLEETSIZE(  2,   5) HAS RANGE OF  1
 FLEETSIZE(  3,   5) HAS RANGE OF  1
 FLEETSIZE(  4,   5) HAS RANGE OF  2
 FLEETSIZE(  5,   5) HAS RANGE OF  2
 FLEETSIZE(  6,   5) HAS RANGE OF  3
 FLEETSIZE(  7,   5) HAS RANGE OF  3
 FLEETSIZE(  8,   5) HAS RANGE OF  3
 FLEETSIZE(  9,   5) HAS RANGE OF  4
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FLEETSIZE(  6,  4) HAS RANGE OF  4
FLEETSIZE(  7,  4) HAS RANGE OF  4
FLEETSIZE(  8,  4) HAS RANGE OF  4
FLEETSIZE(  9,  4) HAS RANGE OF  4
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FLEETSIZE(  2,  3) HAS RANGE OF  2
FLEETSIZE(  3,  3) HAS RANGE OF  4
FLEETSIZE(  4,  3) HAS RANGE OF  4
FLEETSIZE(  5,  3) HAS RANGE OF  4
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FLEETSIZE(  7,  3) HAS RANGE OF  4
FLEETSIZE(  8,  3) HAS RANGE OF  4
FLEETSIZE(  9,  3) HAS RANGE OF  4
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FLEETSIZE(  2,  2) HAS RANGE OF  3
FLEETSIZE(  3,  2) HAS RANGE OF  4
FLEETSIZE(  4,  2) HAS RANGE OF  4
FLEETSIZE(  5,  2) HAS RANGE OF  4
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FLEETSIZE(  7,  2) HAS RANGE OF  4
FLEETSIZE(  8,  2) HAS RANGE OF  4
FLEETSIZE(  9,  2) HAS RANGE OF  4
FLEETSIZE(  1,  1) HAS RANGE OF  1
FLEETSIZE(  2,  1) HAS RANGE OF  3
FLEETSIZE(  3,  1) HAS RANGE OF  4
FLEETSIZE(  4,  1) HAS RANGE OF  4
FLEETSIZE(  5,  1) HAS RANGE OF  4
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FLEETSIZE(  7,  1) HAS RANGE OF  4
FLEETSIZE(  8,  1) HAS RANGE OF  4
FLEETSIZE(  9,  1) HAS RANGE OF  4
```

BUS FLEETSIZE COST MATRIX

STATE/STAGE	6	5	4	3	2	1
1	1846500.	1455750.	228850.	6190500.	1192200.	2287090.
2	1846500.	1455750.	228850.	5345500.	1021150.	1970345.
3	1846500.	1455750.	128900.	4974250.	973600.	1880190.
4	1495250.	904750.	128900.	4974250.	973600.	1880190.
5	1495250.	904750.	128900.	4974250.	973600.	1880190.
6	1495250.	904750.	128900.	4974250.	973600.	1880190.
7	1495250.	904750.	128900.	4974250.	973600.	1880190.
8	1495250.	904750.	128900.	4974250.	973600.	1880190.
9	1495250.	904750.	128900.	4974250.	973600.	1880190.

```
                   CALCULATION ======>

          PATH MATRIX
 STATE/STAGE  1  2  3  4  5
   1          1  1  1  1  1
   2          2  2  2  2  2
   3          3  3  3  3  3
   4          4  4  4  4  4
   5          5  5  5  5  5
   6          6  6  6  6  6
   7          7  7  7  7  7
   8          8  8  8  8  8
   9          9  9  9  9  9


 * * * * * * * * * * * BUS SYSTEM STUDY RESULTS * * * * * * * * * * *


 PROPOSED BUS ROUTE NOT RECOMMENDED
 **FORTRAN ** STOP
%/ENDP
/
```

```
DO LEE.LEEDP.PROC
%/PROC C
%/FILE LEE.LEEDP.INPUT,LINK=DSET70,FCBTYPE=ISAM,RECFORM=V
%/EXEC LEEDP
% C P500 LOADING.
FORTRAN IV PROGRAM LEEDP   STARTED --- 09/28/73
```

DYNAMIC PROGRAMMING MODEL

YOUNG LEE

INPUT DATA

```
M=  4                DELTA= 50
THETA=  5            NSTAGE=  6
NSTATE=  9           OWC= $   5000.00
```

STAGE	DELTA/THETA	STAGE FACTOR
6	12.00	0.70
5	12.00	0.70
4	15.00	0.80
3	18.00	0.85
2	27.00	0.95
1	27.00	1.00

BUS FREQUENCY COSTS

STATE/STAGE	6	5	4	3	2	1
1	36930.	29114.	4576.	123810.	23844.	47740.
2	29904.	18094.	2578.	106910.	21960.	41670.
3	30000.	19120.	2926.	99484.	20422.	39406.
4	30000.	20144.	3274.	101224.	19472.	37602.

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FLEETSIZE(  8,   6) HAS RANGE OF  4
FLEETSIZE(  9,   6) HAS RANGE OF  4
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FLEETSIZE(  9,   5) HAS RANGE OF  4
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CALCULATION ======:

PATH MATRIX

STATE/STAGE	1	2	3	4	5
1	2	2	1	2	1
2	2	2	2	2	2
3	3	3	3	3	3
4	4	4	4	4	4
5	5	5	5	5	5
6	6	6	6	6	6
7	7	7	7	7	7
8	8	8	8	8	8
9	9	9	9	9	9

* * * * * * * * * * * BUS SYSTEM STUDY RESULTS * * * * * * * * * * *

SCHEDULE PERIOD	FLEET SIZE
SUN	5 BUSES
SAT OFF	5 BUSES
WEEK OFF	5 BUSES
SAT PEAK	5 BUSES
A.M. PEAK	5 BUSES
P.M. PEAK	5 BUSES

SCHEDULE PERIOD	SERVICE FREQUENCY
SUN	100 DISPATCHES
SAT OFF	100 DISPATCHES
WEEK OFF	50 DISPATCHES
SAT PEAK	50 DISPATCHES
A.M. PEAK	50 DISPATCHES
P.M. PEAK	50 DISPATCHES

OPTIMUM FLEET SIZE OF PROPOSED BUS ROUTE IS 5 BUSES

OPTIMUM TOTAL BUS TRANSIT SYSTEM COST IS $ 242314.00

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